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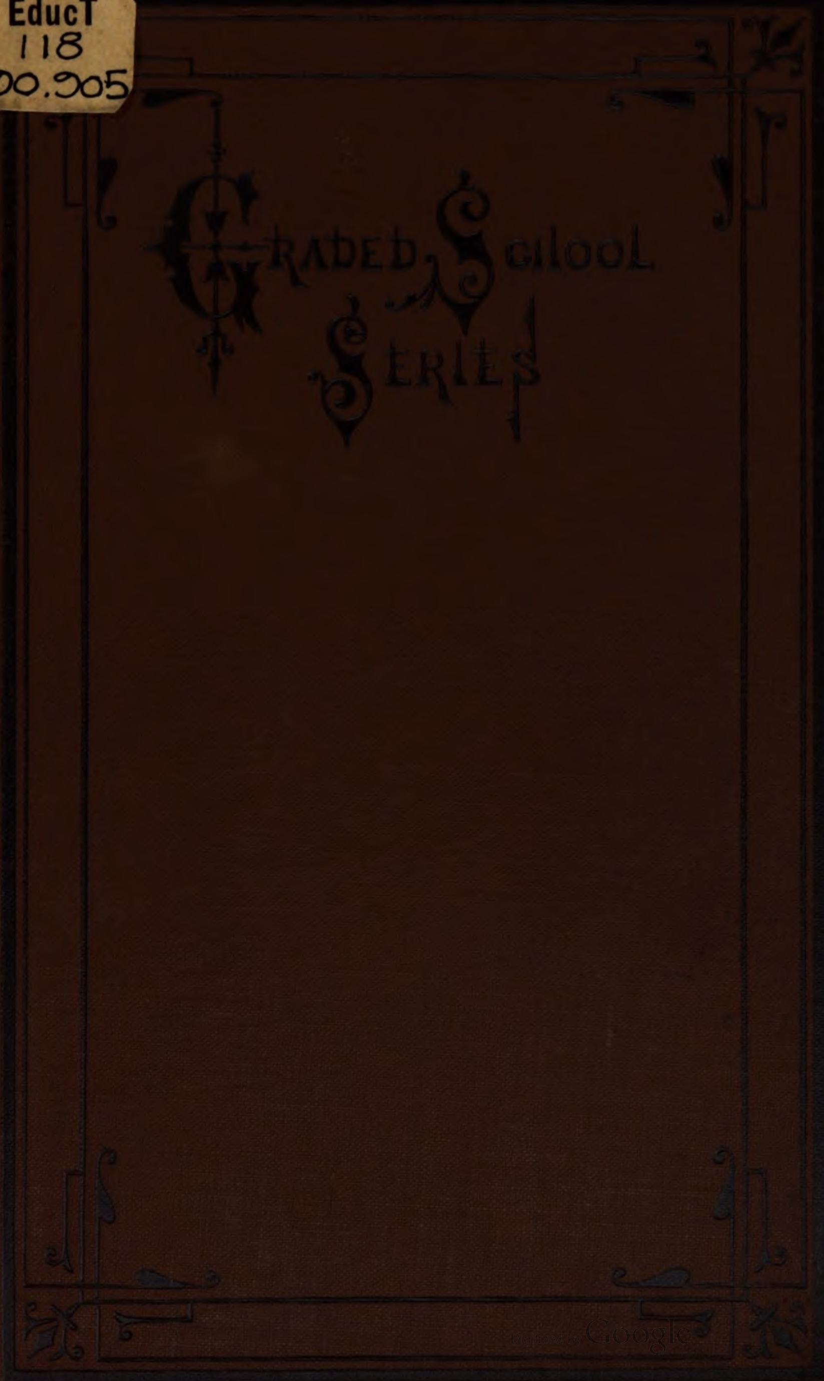

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A
MANUAL OF ARITHMETIC

CONTAINING

SUGGESTIONS, METHODS OF TEACHING, MODELS
OF ANALYSIS, AND ILLUSTRATIVE
SOLUTIONS OF PROBLEMS.

A Hand-book for Teachers.

BY

E. E. WHITE, A. M.,

Author of Graded-School Arithmetics, School Registers and Records, etc.

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PREFACE.

THE author has long contemplated the issuing of a manual of methods and suggestions, to accompany the Graded-School Arithmetics, but circumstances beyond his control have prevented its preparation until the present year.

The MANUAL now presented is specially designed for those teachers who desire to improve their methods of teaching arithmetic, and it is hoped that it may prove as useful as its plan is unique.

Part I is chiefly devoted to the teaching of the more elementary processes. The highest arithmetic in the series is, as a rule, taught better than the lower, and there is less demand for methods and suggestions. There is an earnest inquiry among teachers for approved methods of elementary instruction.

Part II contains illustrative solutions of all the more difficult problems in the two higher books. The solutions of the mental problems are presented as models of analysis, and the solutions of the written problems are designed to be illustrations of slate and blackboard work. In the solutions of both classes of problems, great pains has been taken to present the best possible forms. The explanations added to the solutions of the written problems present, in most cases, what should be given orally by the pupil in the recitation.

The author is indebted to A. G. Farr, of the Columbus (O.) High School for valuable assistance in the preparation of Part II.

LAFAYETTE. IND., *May*, 1876.

CONTENTS.

PART FIRST.

PAGE
Methods of Teaching Arithmetic 9

THE PRIMARY ARITHMETIC.

General Plan 15
Numbering Groups of Objects 16
Addition and Subtraction with Numbers not exceeding Ten . 18
Addition and Subtraction with Numbers not exceeding Twenty 22
Tables 20, 24, 33, 35, 38
Blackboard Drills 25, 33, 37, 38
Analysis of Numbers 26
Numbers from Twenty to One Hundred 27
Addition 30
Subtraction 32
Multiplication 34
Division 37
Review of the Four Operations 39
Oral Lessons in Fractions 41
Oral Lessons in Denominate Numbers 43

THE INTERMEDIATE ARITHMETIC.

General Suggestions 53
Notation and Numeration 56
Addition 57
Subtraction 61
Multiplication 62
Division 63
Drills in Rapid Combination 65
Common Fractions 67

PAGE
Decimal Fractions 69
United States Money 73
Denominate Numbers 74
Percentage 76
Drill Problems 78

THE COMPLETE ARITHMETIC.

General Suggestions 97
An Abridged Course 98
Abbreviated Processes 98

PART SECOND.

THE INTERMEDIATE ARITHMETIC.

Models of Analysis and Illustrative Solutions 3

THE COMPLETE ARITHMETIC.

Models of Analysis and Illustrative Solutions . . . 11 to 115
  Fractions 11
  Mensuration 16
  Denominate Numbers 18
  Compound Numbers 20
  Percentage 23
  Interest 38
  Equation of Payments 60
  Proportion 66
  Partnership 68
  Problems for Analysis 70
  Evolution 85
  General Review 90
  Appendix 110

PART I.

METHODS OF TEACHING ARITHMETIC, WITH SUGGESTIONS, EXPLANATIONS, ILLUSTRATIVE LESSONS, DICTATION EXERCISES, AND DRILL PROBLEMS.

METHODS OF TEACHING ARITHMETIC.

The Rule Method.

THE old method of teaching Arithmetic required the pupil first to commit the definitions and the rule, and then to solve the problems "according to the rule" and with constant reference to it. This is known as the *Rule Method.* Its obvious defects, both as a means of mental training and as a preparation for practical business, caused it long since to be discarded by the best teachers. Its continued use in many schools is chiefly due to the fact that most of the arithmetics, lately in common use, give first the definitions, then the rule, and then the problems, and this order of the text-book is too often blindly followed by teachers. When the text-book prefaces the rule by the solution of one or two problems, the rote teacher either ignores the solutions or uses them as mere illustrations of the rule. No attempt is made to derive the rule from the given solutions.

The Analytic Method.

In marked contrast with the rule method is the solution of arithmetical problems by logical analysis and without reference to any rule or principle. This is known as the *Analytic Method.* For years it was chiefly used in an independent course of instruction, called "Mental Arithmetic."

The method has great value as a means of mental training and also of imparting accuracy and rapidity in numerical computations, but, when confined to the oral solution of problems, it neither teaches the principles of numbers nor prepares the pupil for their easy comprehension. It has often been observed that pupils who have been most persistently drilled in mental arithmetic show little increased ability to understand mathematical principles and as little increased skill in their statement or application.

This result may be partly due to the fact that the logical analyses of problems are often memorized by pupils without being comprehended, thus injuring, instead of improving, the reasoning faculty. The chief source of the difficulty, however, is in making analysis a complete and independent method of arithmetical instruction. Analysis is but one step in a complete method. The results obtained by it need to be compared and generalized, thus reaching general processes and principles.

The Inductive Method.

Processes and principles are best taught by the *Inductive Method.* Written processes may be derived by easy inductions from the oral analyses, and both oral and written processes may thus be united *as the essential complements of each other.* The first step is the oral analysis of problems involving numbers small enough to be easily held in the memory, and when this step is well taken, the pupil is prepared for the written process. When, for example, a pupil can find $\frac{2}{3}$ of $\frac{5}{6}$ analytically, he will have little difficulty in deducing a general written process for finding a fraction of a fraction. The pupil who can show by analysis that 5 is ten per cent of 50, has the key to the written process for finding what per cent one number is of another. It is evident that a process thus learned will be better understood and longer remembered than is possible when the oral analysis is omitted. Processes learned mechanically may be soon forgotten, since the reason does not assist the memory.

But arithmetical instruction should not *end* with the solution of problems. It should also teach the *science* of numbers. The written processes should be analyzed, and descriptive statements of the successive steps united in a concise and clear generalization in the form of a rule. Principles and definitions should in like manner be deduced and formulated, and definitions, principles, and rules, thus reached, should be committed to memory.

The Natural Method.

The union of the analytic and the inductive methods, as above indicated, may properly be called the *Natural Method* of teaching arithmetic. It conforms to a natural order. It not only teaches "processes before rules," but "rules *through* processes," thus observing two important maxims in a natural system of instruction. It teaches arithmetic both as an art and as a science, and imparts a practical knowledge of both processes and principles.

This Natural Method is completely embodied in the *Graded-School Arithmetics*. Oral and written processes are combined in each book of the series, and this union is natural and complete. The mental problems are preparatory to the written which accompany and supplement them. Nor are the oral solutions introduced as mere illustrations of the written, but the mental problems are sufficiently numerous and varied to enable the pupil to master the analytic processes before he attempts to derive therefrom the written. The mental problems in this series of arithmetics furnish as varied and comprehensive drills in oral analysis as those in any one of the standard mental arithmetics. These mental problems may also be used as written problems, thus increasing the amount of written work, and making the induction of the written process an easy step for the pupil. The importance of this suggestion justifies the following direction to teachers:

In the use of the Graded-School Arithmetics, the mental problems in each lesson or article should be solved first orally and then in a written form, the written process being derived from the oral analyses. The written problems may next be solved.

These arithmetics not only combine the analytic and inductive methods in the processes, but the rules and principles, and most of the definitions, are placed *after* the problems, and are derived inductively from the solutions. This is a characteristic feature of this pre-eminently *Inductive Series of Arithmetics.*

In another place (page 59) the method of deriving the rule from the process is fully explained. It must suffice to say here that the written process is to be generalized in the form of a rule *by the pupil,* and only *when he is prepared for the induction.* This preparation may require the solution of all or only a few of the written problems. The written process must be understood as a process before it can be intelligently generalized.

It is thus seen that there is no advantage in placing the author's rule after say four to six problems, since no author can tell how many solutions will be required by the pupil. Moreover, the author's rule is not needed *until the pupil has made a rule,* and then the author's rule should be used as a model for the improvement of the pupil's. It is one thing for a pupil to see how the author has generalized a process, and quite another for the pupil to make a rule by a proper generalization. Besides, the placing of the author's rule before all or most of the problems results in the pupil's solving the problems *by the rule,* and thus the old rule method, with its unsatisfactory results, is perpetuated in the schools. Pupils need to be trained to discover the solution of an arithmetical problem by an analysis of its conditions, and this training can not be secured by the mechanical solution of problems under and according to a given rule.

Even in those applications of numbers which are determined by business custom or by law, such as the applications of percentage, the process may be taught as a process more readily than it can be taught through the rule.

A correct method of arithmetical instruction requires the author's rules to be placed after the problems, to be used when the process has been mastered and generalized by the pupil.

All definitions, principles, and formulas, which can be taught inductively, are placed after the problems in this series of arithmetics. When definitions can not be taught inductively, as in the applications of percentage, they properly precede the problems. These definitions usually relate to terms employed in the statement of the problems, and hence they must be understood before the problems can be understood or intelligently solved. The definitions used in mensuration must be taught by geometry, and hence they may precede the problems. The same is true of those rules of mensuration which are used mechanically in the solution of problems. It is here that the rule method has its place.

The natural method also requires the instruction in arithmetic to be adapted, both in matter and method, to the capacity and attainments of the pupil. This principle is carefully embodied in the Graded-School Arithmetics. The successive books are specially adapted to the grade of pupils for which they are intended, and each contains those processes and principles which should be mastered by the pupil before the study of the next higher book is begun. This careful gradation permits a fuller and more progressive treatment of the several subjects than is possible when the lower books are mere abridgments of the higher. It also makes the lower books an excellent shorter course in arithmetic for those pupils who do not remain in school long enough to take a full course. Many pupils, both in country and city, leave school before they reach the Complete Arithmetic, or other arithmetic of like grade, and many more leave school before they are prepared to study such an arithmetic. What they need is a text-book presenting the elementary and practical processes of arithmetic *in a manner adapted to their limited attainments and opportunities.* They have neither capacity nor time to master an arithmetic designed for more advanced pupils. The recent addition of the more useful processes in Percentage and Mensuration to the Intermediate Arithmetic fully adapts it to the wants of this class of pupils. It presents a SHORT COURSE IN ARITHMETIC, with thorough drills in all elementary processes and with a brief treatment of those practical applications which are most frequently used in business.

The natural method of teaching arithmetic, above sketched, has been widely adopted by the best teachers, and with most satisfactory results. Both reason and experience unite in commending it as the true method.

It may be added that arithmetics embodying the natural method, are also adapted to other methods. The mental problems can be analyzed without reference to the written processes, thus separating what the author has wisely joined together. The pupils may be required to begin with the rule and solve the problems by it, and definitions, principles, and rules may be learned by rote. The results thus attained will be as satisfactory as those secured by the use of books based on these methods or on no method.

THE PRIMARY ARITHMETIC.

General Plan.

The aim of the Primary Arithmetic is to teach young children to read, write, add, subtract, multiply, and divide small integral numbers with accuracy and rapidity. All the fundamental operations of numbers in the decimal system are based on the combination of the integers from one to nine inclusive, and hence the combination of these digital numbers is the first step in a natural course of arithmetical instruction.

The greatest result obtained by combining any two digital numbers is 81 (9 × 9), and hence these primary operations can all be taught by the use of numbers expressed by one and two figures—numbers within the easy comprehension of the young pupil. Until the child has learned to combine the digital numbers readily, he is not prepared to read, write, and combine numbers composed of several periods, much less to understand the *theory* of the decimal system. THE FIRST LESSONS IN ARITHMETIC SHOULD BE CONFINED TO PRIMARY PROCESSES WITH SMALL NUMBERS.

The first nineteen lessons in the Primary Arithmetic involve no number which exceeds *ten*, the next twenty-two lessons involve no number which exceeds *one hundred*, the next forty-five lessons involve no number which exceeds *one thousand*, and only the last three lessons contain numbers which exceed one thousand. With these small numbers a great number and variety of progressive exercises are provided—exercises which may be mastered by children at an early age. To this end the drills must be *thorough*, each step being *repeated* by the pupil until it can be taken without hesitation. No lesson is properly mastered until it can be recited as promptly as familiar lessons in reading and spelling. The result of a given combination of numbers must be seen as readily as a known word is recognized. The primary processes in numbers may thus be mastered in the first two years of the child's school course, and the fact that this is not usually accomplished in even five years, explains the slow progress of pupils in arithmetic.

The first twenty lessons may be taught ORALLY *to children who can not read well enough to use the text. The youngest pupils can, however, use the pictures, tables, and written exercises. The lessons should be carefully studied by the teacher as models of oral teaching.*

LESSONS I TO X.

Numbering Groups of Objects.

The object of these ten lessons is, (1) to develop the idea of the numbers from one to ten, and, (2) to teach the child to number the objects in a group, containing not more than ten, *without counting*. Very few pupils enter school without having an idea of each number from one to ten, but nearly all enter without the ability to recognize the number of objects in groups of one to ten *at sight*. The great majority of primary pupils can not give at sight the number of objects in groups of more than three or four. Hence the *chief* aim of these lessons is to develop this *perceptive power*, as a preparation for the succeeding lessons in addition and subtraction; and the training which will best secure this result, will, *at the same time*, develop the idea of these numbers and impart the ability to count them. But little attention need be given in school instruction to *counting by ones*, and CHILDREN SHOULD NEVER BE TAUGHT TO ADD AND SUBTRACT BY COUNTING. The habit of counting must be overcome before the pupil can acquire the art of rapid computation. Counting is not adding, and the majority of pupils who once learn to combine numbers by counting, never learn to add.

In giving these ten lessons orally, the teacher should use small objects, as the fingers, blocks, buttons, pen-holders, match-sticks, etc. The blackboard should also be used, but care should be taken to prevent the *counting of the marks, letters, etc., as they are made.* This counting may be prevented by making several groups of ten characters on the blackboard, and then covering portions of the groups successively and having the pupils give the number *in sight.* The drill must be progressive, as indicated in the book, beginning with small groups, and gradually introducing larger. A variety of objects should be used, and these should be handled by the pupils, as well as by the teacher.

Written Exercises.

Each lesson properly closes *with a written or slate exercise.* The eye and the hand are thus trained together, and oral and written exercises are united *from the first.* The laws of child activity demand a variety of exercise, both of mind and body, and it is a violation of these laws to keep a child counting for days, then making figures, then adding, etc. The maxim, "one thing at a time," is often sadly abused in teaching little children. In primary instruction the "time" of one thing is a very short period. One thing is to be done only so long as the attention can be held and the interest secured, and then another thing must be done. A better maxim for the primary teacher is, "One thing at a time, but several things in an hour." The figures should be made in their *simplest forms*, and special pains should be taken to show how each one is made. This can best be done by making the figure on the board, and, at the same time, describing the process in words, to be repeated by the pupils. The forms of the letters should be shown by familiar illustrations. The work of the pupils on their slates should be inspected by the teacher, and the more common errors shown on the blackboard. Their correction will require additional practice.

THE "LESSONS," INTO WHICH THE PRIMARY ARITHMETIC IS DIVIDED, ARE NOT INTENDED TO BE GONE OVER IN ONE EXERCISE EACH, OR IN ONE DAY. It will require several days to teach thoroughly any one of them,—the first four or five excepted. In order to make the most rapid progress, the primary teacher should "make haste slowly."

LESSONS XI TO XX.

Addition and Subtraction with Numbers not exceeding Ten.

The object of these lessons is to teach the addition and subtraction of numbers with amounts and minuends not exceeding ten. The previous drill in numbering groups of objects has prepared the pupil for the easy mastery of these lessons *without counting.*

The first step is the adding and subtracting of groups of objects *in sight;* the second step is the adding and subtracting of groups of objects *not in sight*—the adding and subtracting of *concrete numbers;* and the third step is the adding and subtracting of the corresponding *abstract numbers.* This natural order is faithfully observed in the lessons, the three steps being united in reaching each combination.

In giving these lessons the teacher should use small objects, as in the previous lessons, and great care should be taken not to reach the third step—the combination of abstract numbers—before the pupil can readily take the other two steps. Exercises in adding groups of objects, in sight and out of sight, should be multiplied until the pupil can give the result *instantly;* and the adding and subtracting of abstract numbers should be reached as a final step. "From the concrete to the abstract" is the guiding maxim.

THE VERY COMMON ERROR OF GIVING ALMOST EXCLUSIVE ATTENTION TO DRILLS ON ABSTRACT NUMBERS, SHOULD BE CAREFULLY AVOIDED IN THESE FIRST LESSONS.

Addition and Subtraction United.

In these lessons addition and subtraction are united as inverse operations. This is a natural union, and experience proves that both processes may be thus taught together more readily than they can be taught separately. When the pupil has combined two groups of objects and found their sum, it is a natural and easy step to take one of the groups away to see how many are left. The analysis supplements and aids the synthesis. Nor does the union of the two processes cause the least confusion in the minds of children. Children's minds are not as easily confused as the minds of some older people who theorize on primary teaching!

If, however, any teacher prefers to teach one process at a time, the exercises in subtraction can be omitted until Lesson XLIII is reached, when they may be given as preparatory drills. See page 32.

Oral Analysis.

No attempt should be made to teach the analysis of any of the examples in the first thirty lessons of the Primary Arithmetic, and nothing will be gained by requiring the pupil to give *reasons* for processes or results.

The solutions of a few of the review examples in Lesson XIX are here given as models:

1. 8 fingers; 5 fingers and 3 fingers are 8 fingers.
2. 2 blocks; 7 blocks from 9 blocks leave 2 blocks.
5. 9 cows; 5 cows and 4 cows are 9 cows.
6. 3 cents; 7 cents from 10 cents leave 3 cents.
7. 3 peaches; 6 peaches from 9 peaches leave 3 peaches.

They may also be solved thus:

1. 5 fingers and 3 fingers are 8 fingers; they both hold up 8 fingers.
2. 7 blocks from 9 blocks leave 2 blocks; 2 blocks will be left on the table.
5. 5 cows and 4 cows are 9 cows; he has 9 cows in both lots.
6. 7 cts. from 10 cts. leave 3 cts.; he had 3 cts. left.
7. 6 peaches from 9 peaches leave 3 peaches; she had 3 peaches left.

The first form is the simpler for young children.

Tables in Addition and Subtraction.

In most elementary arithmetics these tables *precede* the exercises and are given in a complete form, to be memorized by the pupil—a plan in harmony with the old rote method of teaching. In this book the tables follow the three-step exercises above described, and they are given in an *incomplete form*, the results being omitted, to be supplied by the pupil. The pupil is first *to make the table*, and *then to memorize it*, and the previous drill makes both steps easy.

The table in each lesson should be copied on the blackboard in its incomplete form, for oral drill, the pupils being required to find out and give the sum or difference, as the case may be. The table, on page 19, may be recited thus, the teacher designating the combinations with a pointer: 1 and 2 are 3; 2 and 2 are 4; 3 and 2 are 5, etc.

The pupils should also be required to copy and complete the tables on the slate in this form:

0 and 2 are 2
1 “ 2 “ 3
2 “ 2 “ 4
3 “ 2 “ 5
4 “ 2 “ 6
5 “ 2 “ 7
6 “ 2 “ 8
7 “ 2 “ 9
8 “ 2 “ 10

2 from 2 leaves 0
2 “ 3 “ 1
2 “ 4 “ 2
2 “ 5 “ 3
2 “ 6 “ 4
2 “ 7 “ 5
2 “ 8 “ 6
2 “ 9 “ 7
2 “ 10 “ 8

The blackboard drill on the tables may precede or follow the copying and completion of the tables on the slate by the pupils. With many classes it may be well to have a drill on the blackboard both before and after the slate exercise. As a final step the pupils should be required *to repeat the tables rapidly from memory*, without the aid of slate or blackboard.

In making these tables on the slate or blackboard, the signs +, —, and = may be used, but it is believed that there is nothing gained by their use *at this early stage in the pupil's progress.* It is better to use the same language in the tables and in the exercises, and to combine the numbers *in the same order.* The first point may be secured by having the sign + read “and;” = “are” or “is;” and — “less;” but + does *not* mean “and,” and = does not mean “are” or “is,” and it is an error so to read them. In taking a group of 2 blocks from another of 7 blocks, the order is 2 blocks from 7 blocks leave 5 blocks; but when the signs — and = are used, the order of minuend and subtrahend is reversed, thus: 7 blocks — 2 blocks = 5 blocks. This difficulty may be removed by using the word “less” in the exercises.

When inverted commas (“) are used, as above, the tables can be made as rapidly as when signs are used.

Written Exercises.

The written exercises at the close of each lesson should be copied by the pupils on their slates, with the results written below. These and other similar exercises should also be written on the blackboard and used for class drills. The drills should be continued until the results are given instantly. In adding columns composed of more than two figures, it may be well *at first* to permit the pupils to name the two lower numbers and give their sum, then name this sum and the next number and give their sum, and so on, until the process is understood. The pupils should then be taught to add rapidly, *naming only the successive amounts.* The column at the left should, for example, be added thus: 3, 5, 7, 8, 10. Prove by adding downward.

$$\begin{array}{r} 2\\ 1\\ 2\\ 2\\ 1\\ \underline{2}\\ 10 \end{array}$$

LESSONS XX TO XXVIII.

Addition and Subtraction with Numbers not Exceeding Twenty.

The preceding lessons are limited to numbers not exceeding ten, to secure requisite skill in adding the smaller numbers, 1, 2, 3, 4, and 5, before using 6, 7, 8, and 9 as additive numbers. The object of these eight lessons is to teach the addition and subtraction of all the digital numbers, with amounts and minuends not exceeding twenty.

Numbers from Ten to Twenty.

The first two lessons, XX and XXI, develop the idea of each number from ten to twenty, and teach its name and its representation by figures. The method of reaching each result is fully presented in the lessons, where the idea, the name, and the representation of each number are taught together. In teaching very young pupils, it may be better to omit the representation by figures until the idea and name of each number from ten to fifteen have been taught. The analysis of the words *thirteen*, *fourteen*, *fifteen*, *sixteen*, etc., may best be given at the close of the lesson.

In developing an idea of the numbers eleven, twelve, thirteen, etc., do not require the pupils to *count* from one to eleven, one to twelve, one to thirteen, etc., but begin with the number *ten*, the base of the decimal system, and develop each higher number by *adding the requisite number to ten*. Eleven is *ten* and *one;* twelve is *ten* and *two;* thirteen is *ten* and *three*, etc. Use objects, as in preceding lessons. Take ten sticks (pen-holders or match-sticks) and tie them together in a package, calling it *ten sticks* or simply *ten*. With the package in right hand and one stick in left hand, ask, How many sticks in my right hand? "Ten." How many sticks in my left hand? "One." Bring the hands together, and ask, How many in both? If no pupil answers, give the name of the number, *eleven*. How many sticks are ten sticks and one stick? "Eleven." Put ten blocks in a group on the table, and, adding one block, ask, How many blocks are ten blocks and one block? "Eleven blocks." Make ten marks on the blackboard, inclosing them with curves, and add another mark, thus: (//////////) /, and ask, How many marks have I made? "Eleven." How many marks are ten marks and one mark? "Eleven marks." Let the pupils hand you eleven sticks (using the ten package), eleven blocks, etc.

Teach twelve by adding two sticks to the ten package, two blocks to the ten blocks, two marks to the ten marks, etc., and proceed in this manner to the number fifteen.

Review the preceding exercises and teach the pupils to write eleven, twelve, thirteen, etc., in figures. Show that the 1 in ten denotes one ten (the package of sticks, the group of ten blocks, the ten marks), and that the 0 shows that nothing is added to the ten. Write eleven in figures, 11, and show that the left-hand 1 denotes ten (the package, etc.,) and that the right-hand 1 denotes the one that is added (one stick, etc.). Write twelve in figures, 12, and ask, What does the 1 denote? "Ten." What figure denotes the number added to the ten? "2." Take the ten package and ask a pupil to hand you the number of sticks denoted by the figure 2. Teach in a similar manner the writing of 13, 14, and 15. The idea, the name, and the mode of writing the numbers 16, 17, 18, and 19 may be taught together.

In teaching the meaning of the word *thirteen*, say that the "teen" means *ten*, and that the "thir" means *three*, and hence that "thirteen" means *three* and *ten*—the "and" being understood. Do not teach the error that "teen" means "*and ten.*" The "and" is not in the word thirteen. Teach in a similar manner the meaning of *fourteen*, *fifteen*, *sixteen*, etc., showing that the "four," "fif" (five), "six," etc., denote, respectively, *the number added to ten.*

Tables.

Precede Lesson XXII by drills on the addition tables, with 2, 3, and 4 as the added numbers. The tables may be written on the blackboard in incomplete form, thus:

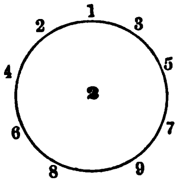
0 and 2	0 and 3	0 and 4
1 " 2	1 " 3	1 " 4
2 " 2	2 " 3	2 " 4
3 " 2	3 " 3	3 " 4
4 " 2	4 " 3	4 " 4
5 " 2	5 " 3	5 " 4
6 " 2	6 " 3	6 " 4
7 " 2	7 " 3	7 " 4
8 " 2	8 " 3	8 " 4
9 " 2	9 " 3	9 " 4
10 " 2	10 " 3	10 " 4

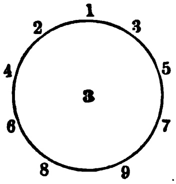
Have each table recited forwards and backwards, the pupils saying 0 and 2 are 2; 1 and 2 are 3; 2 and 2 are 4, etc., and 10 and 2 are 12; 9 and 2 are 11; 8 and 2 are 10, etc. The teacher should also point promiscuously to the numbers to be added, the pupils reciting as before. The drill on each table should be continued until the pupils add the numbers *instantly*. The corresponding tables in subtraction may also be recited.

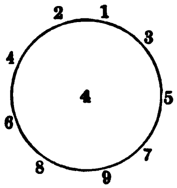
The tables should be written on the slates in a *completed* form. See directions on page 21.

Blackboard Drills.

Blackboard exercises to supplement the above drills, may be thus arranged:







The teacher points to the numbers without the circle and the pupils add to these numbers the number in the center, and give the *amount*. These drills may be multiplied to any desirable extent.

Blackboard exercises in subtraction may be arranged in a similar manner. See page 33.

The next six lessons, XXII—XXVIII, may be taught orally in the same manner as lessons XI to XX, already described. See pages 18 to 22, inclusive. The numbers being larger, the combinations are a little more difficult, and more drill may be needed.

Analysis of Numbers.

An analysis of the numbers to twenty will afford an effective review of the preceding lessons. The analysis should begin with groups of objects, to be handled by the pupils when reciting, and end with abstract numbers. The Numeral Frame will be found very convenient.

The oral analysis of the abstract numbers, from 3 to 10, inclusive, may be made as follows:

3 is 1 and 2, or 2 and 1 : 3 less 1 is 2; 3 less 2 is 1.

4 is 1 and 3, or 3 and 1 : 4 less 1 is 3; 4 less 3 is 1.
4 is 2 and 2, - - - - : 4 less 2 is 2; - - - - -

5 is 1 and 4, or 4 and 1 : 5 less 1 is 4; 5 less 4 is 1.
5 is 2 and 3, or 3 and 2 : 5 less 2 is 3; 5 less 3 is 2.

6 is 1 and 5, or 5 and 1 : 6 less 1 is 5; 6 less 5 is 1.
6 is 2 and 4, or 4 and 2 : 6 less 2 is 4; 6 less 4 is 2.
6 is 3 and 3, - - - - : 6 less 3 is 3; - - - - -

7 is 1 and 6, or 6 and 1 : 7 less 1 is 6; 7 less 6 is 1.
7 is 2 and 5, or 5 and 2 : 7 less 2 is 5; 7 less 5 is 2.
7 is 3 and 4, or 4 and 3 : 7 less 3 is 4; 7 less 4 is 3.

8 is 1 and 7, or 7 and 1 : 8 less 1 is 7; 8 less 7 is 1.
8 is 2 and 6, or 6 and 2 : 8 less 2 is 6; 8 less 6 is 2.
8 is 3 and 5, or 5 and 3 : 8 less 3 is 5; 8 less 5 is 3.
8 is 4 and 4, - - - - : 8 less 4 is 4; - - - - -

9 is 1 and 8, or 8 and 1 : 9 less 1 is 8; 9 less 8 is 1.
9 is 2 and 7, or 7 and 2 : 9 less 2 is 7; 9 less 7 is 2.
9 is 3 and 6, or 6 and 3 : 9 less 3 is 6; 9 less 6 is 3.
9 is 4 and 5, or 5 and 4 : 9 less 4 is 5; 9 less 5 is 4.

10 is 1 and 9, or 9 and 1 : 10 less 1 is 9; 10 less 9 is 1.
10 is 2 and 8, or 8 and 2 : 10 less 2 is 8; 10 less 8 is 2.
10 is 3 and 7, or 7 and 3 : 10 less 3 is 7; 10 less 7 is 3.
10 is 4 and 6, or 6 and 4 : 10 less 4 is 6; 10 less 6 is 4.
10 is 5 and 5, - - - - : 10 less 5 is 5; - - - - -

The numbers 11, 12, 13, 14, 15, 16, 17, and 18, may be analyzed in like manner.

LESSON XXIX.

Numbers from Twenty to One Hundred.

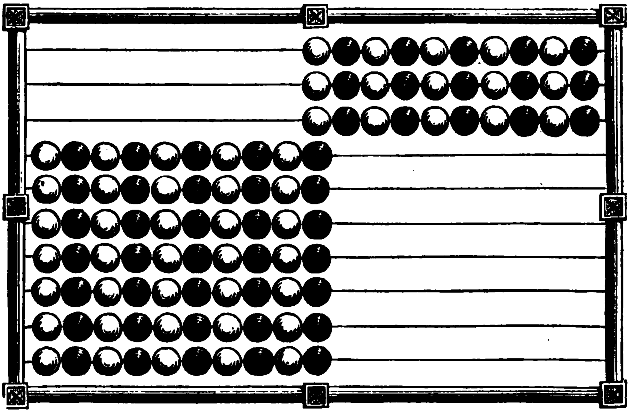
The object of this lesson is to teach numbers *from twenty to one hundred.*

The teacher should be supplied with a numeral frame (see page 28), ten packages of sticks, containing ten each, and ten sticks not tied together, and a blackboard. If a numeral frame can not be obtained, the lessons can be given successfully by means of the sticks and the blackboard. If penholders or match-sticks are not at hand, sticks may be made by cutting twigs into pieces of uniform length (about six inches) and tying them in packages of ten each. Blocks, buttons, and other counters may also be used, but the packages of sticks will be found most convenient.

Take a package of ten sticks in each hand, and ask, How many packages in my left hand? "One." How many sticks in the one package? "Ten." How many packages in my right hand? "One." How many sticks in the one package? "Ten." Bring hands together, and ask, How many packages in both hands? "Two." How many sticks in the two packages? "Two tens." How many sticks in two tens? If no one can answer, say that two tens are called *twenty.* How many sticks in two tens? "Twenty." How many tens in twenty? "Two."

Take two packages in the left hand and one package in the right, and ask, How many packages in my left hand? "Two." How many sticks in the two tens? "Twenty." How many packages in my right hand? "One." How many packages in both hands? "Three." How many sticks in the three packages? "Three tens." How many sticks in three tens? If no pupil can answer, say that three tens are called *thirty*. How many tens in thirty? "Three." How many ones in three tens? "Thirty."

Take the numeral frame, and, after showing that there are ten balls on each wire, slide the upper ten balls to the right or left, and ask, How many balls on this wire? "Ten." Slide the ten balls on the second wire, and ask, How many balls on this wire? "Ten." Slide the ten balls on the third wire, and ask, How many balls on this wire? "Ten." How many ten balls on the three wires? "Three." How many balls on the three wires? "Three tens, or thirty." How many ones in three tens? "Thirty."



Make three rows, of ten marks each, on the blackboard, enclosing each ten marks with curves, as shown below:

(//////////)
(//////////)
(//////////)

How many marks in each row? "Ten." How many tens in the two upper rows? "Two." How many marks in the two upper rows? "Twenty." How many tens in the three rows? "Three." How many marks in the three rows? "Thirty."

Proceed in like manner to teach the number of tens and the number of ones in *forty*, *fifty*, *sixty*, *seventy*, *eighty*, and *ninety*. When the pupils are somewhat familiar with the terms *twenty*, *thirty*, *forty*, *fifty*, etc., call attention to the fact that the second syllable in each ("ty") means *tens*, and that the first syllable denotes *the number of tens*—"twen" meaning *two*; "thir," *three*; "for," *four*; "fif," *five*, etc. Ask, What does *twenty* mean? "Two tens." Thirty? "Three tens." Forty? "Four tens." Fifty? "Five tens."

Writing the Decades in Figures.

The next step is to teach the pupils to write the decades *in figures*. Write several decades on the blackboard in a column, and show that the figure at the left of the naught (0) denotes *the number of tens*. Ask, How many tens in 20? "Two." What figure in 20 denotes the number of tens? "The '2.'" In 40? "The '4.'" In 30? "The '3.'" In 50? "The '5.'"

10
20
30
40
50
60
70
80

The pupils may now be permitted to combine objects in groups of ten each, to make groups of ten marks each, etc., and write *in figures* the number of ones in two groups, three groups, etc.

Writing Numbers between the Decades.

When the decades have been properly taught, it is an easy task to teach pupils to express in words two tens and one (twenty-one), two tens and two ones (twenty-two), etc. Take two packages, of ten sticks each, in the left hand, and adding successively one stick, two sticks, three sticks, etc., ask, How many sticks are two tens? "Twenty." Two tens and one? "Twenty-one." Two tens and two ones? "Twenty-two." Two tens and three ones? "Twenty-three." Two tens and four ones? "Twenty-four." Proceed in like manner to twenty-nine, and then "skip" backwards and forwards.

Next show how 21, 22, 23, 24, etc., are written in figures. Call attention to the fact that the left-hand figure (2) denotes the tens, as above, and that the right-hand figure denotes the number of ones added to the ones in the tens. Write on the blackboard, in a column, 21, 22, 25, 24, 27, etc., and have the pupils show how many *ones* are denoted by each figure. In like manner the numbers between thirty and forty, forty and fifty, etc., may readily be taught. The exercises in the book will furnish additional drills.

LESSONS XXX TO XLI.

Addition to Amounts not Exceeding One Hundred.

The ground covered by the preceding twenty-nine lessons is sufficient for the first year of the course of instruction in numbers, and if these lessons are thoroughly taught, the succeeding lessons will be mastered without difficulty. When a pupil has learned the numbers from one to one hundred, and can add instantly any two digital numbers, a little practice will enable him to add rapidly any columns of figures to amounts not exceeding one hundred. If, for example, the sum of 8 and 7 is known, the sum of 18 and 7, or 38 and 7, or 68 and 7, is readily found.

The lessons in addition are arranged on the same progressive plan as the preceding lessons. The first lesson has 1 and 2 as additive numbers, the second has 3, the third 4, etc., thus securing the requisite drill in adding each of the digital numbers. The lessons in subtraction are arranged on a similar plan.

Written Exercises.

The written exercises should be used both as slate and blackboard exercises, and the columns should be added both up and down. It seems unnecessary to add that both the oral and the written exercises are to be supplemented by exercises given by the teacher. Many other columns of figures may easily be given. For directions for adding these columns, see page 22.

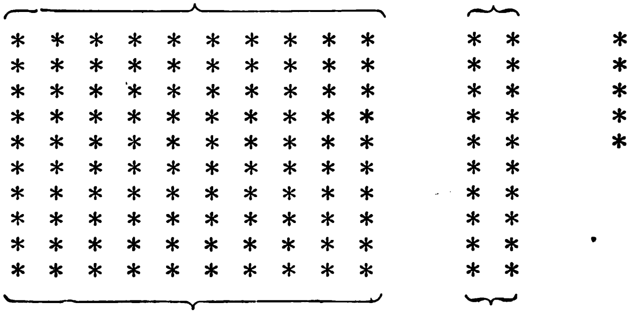
LESSON XLII.

Numbers Expressed by Three Figures.

The object of this lesson is to teach the pupil to read and write any number expressed by three figures.

Show that ten packages, of ten sticks each, tied together, contain *one hundred* sticks. Show how one hundred is expressed in figures (100); also two hundred, three hundred, four hundred, etc.

Take the package of 100 sticks, two packages of 10 sticks each, and 5 sticks, and show that they together contain 125 sticks. Write 125 in figures on the blackboard, and show that the 100 sticks are denoted by the "1," the two tens or 20 sticks by the "2," and the 5 sticks by the "5." The number of ones expressed by each figure in 125 may also be shown on the blackboard, thus:



It will not be necessary to spend much time with counters or with illustrations on the blackboard. The previous lessons have prepared the pupil to understand the value of each of the three figures from an analysis of the abstract number. After showing that the third figure from the right denotes *hundreds*, write on the blackboard 325 in figures, and, pointing to the figure 3, ask, How many *hundreds* are expressed by this figure 3? "Three." How many *ones?* "Three hundred." How many ones are expressed by the figure 2? "Twenty." By the figure 5? "Five." Then how many ones are expressed by the three figures, 325? "Three hundred and twenty-five." Have from ten to twenty numbers analyzed orally in this manner, and then have the same numbers analyzed on the slate, as shown at the right.

$$\begin{array}{r} \underline{325} \\ 300 \\ 20 \\ 5 \end{array}$$

The analysis of numbers orally, and on the blackboard and slate, will enable the pupil not only to read and write any number less than a thousand, but instantly to see the value of each figure.

LESSON XLIII TO LI.

Subtraction.

In the first lessons in the Primary Arithmetic, subtraction is taught as the converse of addition, the two processes being united as complements of each other. In case these exercises in subtraction were omitted (see page 19), they should now be taught as a preparation for lessons XLIII to LI, and it will be well to have all pupils *review*, at this point, the subtraction tables. The drills should be continued until the pupils can instantly subtract any digital number from the sum of it and another digital number; as, 5 from 6 (5 + 1), or 7, or 8, or 9, or 10, or 11, or 12, or 13, or 14; or 8 from 9, or 10, or 11, or 12, or 13, or 14, or 15, or 16.

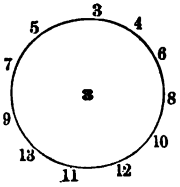
Tables and Blackboard Drills.

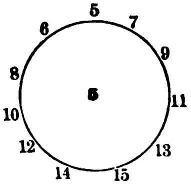
The tables, at this stage, may be written in either of these forms:

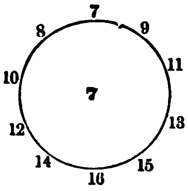
Blackboard.	*Slate.*	*Blackboard.*	*Slate.*
5 less 5	5 less 5 is 0	5 — 5 =	5 — 5 = 0
6 " 5	6 " 5 " 1	6 — 5 =	6 — 5 = 1
7 " 5	7 " 5 " 2	7 — 5 =	7 — 5 = 2
8 " 5	8 " 5 " 3	8 — 5 =	8 — 5 = 3
9 " 5	9 " 5 " 4	9 — 5 =	9 — 5 = 4
10 " 5	10 " 5 " 5	10 — 5 =	10 — 5 = 5
11 " 5	11 " 5 " 6	11 — 5 =	11 — 5 = 6
12 " 5	12 " 5 " 7	12 — 5 =	12 — 5 = 7
13 " 5	13 " 5 " 8	13 — 5 =	13 — 5 = 8
14 " 5	14 " 5 " 9	14 — 5 =	14 — 5 = 9

If the signs — and = are used, teach the pupil to read — *less*, and = *equals*. When the term *minus* is introduced (Primary Arithmetic, p. 107), the pupil may be told that it is a Latin word meaning *less*, and that the sign — may be read either *less* or *minus*, but that it is generally called *minus*.

Blackboard exercises, to supplement the tables, may be arranged as follows:







The pupils are to subtract the number in the center from any number around the circle, to which the teacher points.

The written exercises are to be used both for slate practice and blackboard drills, and they may be multiplied by the teacher until the requisite rapidity is secured.

LESSONS LII TO LXXII.

Multiplication.

These lessons should be preceded by a few oral exercises, designed to develop the idea of multiplication, and to show that two or more times a number is found by adding the number to itself one or more times.

The first step is to show what is meant by once a number, twice a number, three times a number, etc., or by taking a number a given number of times. Speak the word *la* once, and ask, How many times did I say *la?* "Once." How many times do I now say it, *la-la?* "Two times." Now, *la-la-la-la?* "Four times." Now, *la-la-la?* "Three times." Ask the pupils to say *la* once, three times, twice, four times, etc.; to clap the hands once, twice, four times, etc.

Hold up three fingers on the left hand and three fingers on the right hand, and ask, How many times three fingers do I hold up? "Two times." Bring the hands together, and ask, How many fingers are two times three fingers? "Six fingers." Take three blocks and put three other blocks near them, and ask, How many times three blocks are here? "Twice." Slide the two groups together, and ask, How many blocks are twice three blocks? "Six blocks." Make two groups of three marks each on the blackboard, and ask? How many are twice three marks? "Six marks." How many are twice three apples? "Six apples." Twice three boys? "Six boys." Twice three? "Six."

Take three groups, of two blocks each, and, placing them near each other, ask, How many times two blocks are here?

"Three times." How many blocks are three times two blocks? "Six blocks." How many are three times 2 marks? "Six marks." Three times 2 slates? "Six slates." Three times 2? "Six."

Next show that two or more times any number is found *by adding the number to itself.* Twice 2 blocks are 2 blocks and 2 blocks, and twice 2 are 2 and 2; three times 2 blocks are 2 blocks and 2 blocks and 2 blocks, and 3 times 2 are 2 and 2 and 2, etc. This may be shown upon the blackboard, thus:

2 times 2 are 2 and 2.
3 " 2 " 2 and 2 and 2.
4 " 2 " 2 and 2 and 2 and 2.
5 " 2 " 2 and 2 and 2 and 2 and 2.

2 times 3 are 3 and 3.
3 " 3 " 3 and 3 and 3.
4 " 3 " 3 and 3 and 3 and 3.
5 " 3 " 3 and 3 and 3 and 3 and 3.

The sign = may be used in place of the word "are," and the sign + in place of the word "and." If used for the first time, each sign should be explained.

Division may be taught orally in connection with these lessons, as the converse of multiplication. The exercises in division need not be numerous.

The Tables.

The tables in multiplication are *to be made by the pupil* and *then memorized.* The pupil is prepared to do this by the exercises which precede the tables, the number to be multiplied being first represented by pictures of corresponding groups of objects.

The tables in the book are presented in an incomplete form, and, at the right of each table, the number to be multiplied is repeated a corresponding number of times, *to be added by the pupil when this is necessary in order to find or verify the product.* THESE REPEATED NUMBERS SHOULD NOT BE COPIED ON THE BLACKBOARD OR ON THE SLATE. They are not designed to afford practice in adding, but are to be used by the pupil *in preparing or memorizing the table,* when the product is not known and when it can not be found by the pupil *by adding the number to the previous product.* They are not an essential part of the table, and are properly placed *at the right,* to be omitted in reciting, and to be used in study *only when needed,* as above stated.

The tables should be written on the blackboard in an *incomplete* form for class drill, but they should be copied on the slate by the pupils in a *completed* form, thus:

Blackboard	*Slate*
1 time 2	1 time 2 is 2
2 times 2	2 times 2 are 4
3 " 2	3 " 2 " 6
4 " 2	4 " 2 " 8
5 " 2	5 " 2 " 10
6 " 2	6 " 2 " 12
7 " 2	7 " 2 " 14
8 " 2	8 " 2 " 16
9 " 2	9 " 2 " 18

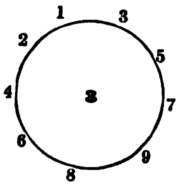
In the tables to be memorized and recited orally, it is better to use the words "times" and "are" or "is" than to use the signs of multiplication (×) and equality (=). When the sign of multiplication is used, the number placed *before* the sign is to be multiplied by the number *after* it; in other words, the number *before* the sign is the *multiplicand* and the number *after* it the *multiplier.* The sign is read "multiplied by;" thus, $3 \times 5 = 15$, is read, 3 *multiplied by* 5 *equals* 15.

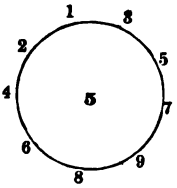
The sign of multiplication may, for convenience, be read "times" if the number *after* the sign is *read first;* thus, 3 × 5 = 15 may be read 5 *times* 3 *equal* 15. It is better to read the sign, in all cases, *multiplied by.*

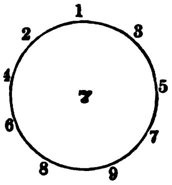
The drills on the tables should be continued until the pupils can give the product of any two digital numbers *instantly,* without adding and without conscious reference to any other product.

Blackboard Drills.

Blackboard exercises, to supplement these drills, may be arranged thus, the *multiplicand* being in the center:







In reciting, the pupils multiply the number at the center by the number without the circle pointed to by the teacher. When, for example, the teacher points to 4 in the left-hand diagram, the pupils say 12.

LESSONS LXXIII TO LXXXIX.

Division.

Division may be taught as a mode of subtraction, but it is more easily taught as the converse of multiplication. In finding how many times 4 is contained in 20, the natural step is to find how many times 4 make 20. Since 5 times 4 are 20, 4 is contained in twenty 5 times.

In these lessons multiplication and division are united as inverse processes, the latter being derived from the former. The true plan is to give the first lessons in division *orally*, in connection with the lessons in multiplication, as suggested in the Primary Arithmetic, page 121. When this has been done, the pupil will find no difficulty in mastering these additional lessons.

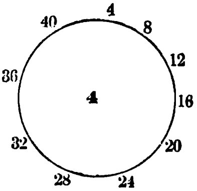
Tables and Blackboard Drills.

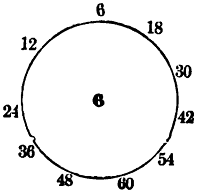
The two connected tables in each lesson are designed for oral and blackboard drills, and they are first to be recited together and then separately. The drills should be continued until the division table can be recited with requisite rapidity.

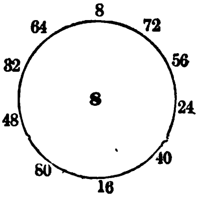
The division table should be written on the slate by the pupils in a completed form, thus:

$3 \div 3 = 1$
$6 \div 3 = 2$
$9 \div 3 = 3$
$12 \div 3 = 4$
$15 \div 3 = 5$
$18 \div 3 = 6$
$21 \div 3 = 7$
$24 \div 3 = 8$
$27 \div 3 = 9$
$30 \div 3 = 10$

Blackboard exercises, to supplement the tables, may be arranged in this form, the *divisor* being placed in the center:







The written exercises can be increased to any desirable extent. It will also be well to require the pupil to test the correctness of the result by multiplying the quotient by the divisor and comparing the product with the dividend. This will assist him in seeing that division and multiplication are inverse processes.

LESSONS XC TO XCII.

Review.

In the preceding lessons addition and subtraction are taught together, and so are multiplication and division. This union of the inverse processes is superior, both in philosophy and practice, to the old method of teaching the four processes separately and in order, and it is also superior to the method that unites the four from the first.

Addition, subtraction, multiplication, and division may, with advantage, be reviewed together. They are united in the oral exercises in the 90th and 91st lessons, and these exercises may be increased to any desirable extent.

The four processes may be readily united in blackboard drills, as follows:

2.	**3.**
$1 + 2 - 2, \times 2 \div 2$	$1 + 3 - 3, \times 3 \div 3$
$2 + 2 - 2, \times 2 \div 2$	$2 + 3 - 3, \times 3 \div 3$
$3 + 2 - 2, \times 2 \div 2$	$3 + 3 - 3, \times 3 \div 3$
$4 + 2 - 2, \times 2 \div 2$	$4 + 3 - 3, \times 3 \div 3$
$5 + 2 - 2, \times 2 \div 2$	$5 + 3 - 3, \times 3 \div 3$
$6 + 2 - 2, \times 2 \div 2$	$6 + 3 - 3, \times 3 \div 3$
$7 + 2 - 2, \times 2 \div 2$	$7 + 3 - 3, \times 3 \div 3$
$8 + 2 - 2, \times 2 \div 2$	$8 + 3 - 3, \times 3 \div 3$
$9 + 2 - 2, \times 2 \div 2$	$9 + 3 - 3, \times 3 \div 3$
$10 + 2 - 2, \times 2 \div 2$	$10 + 3 - 3, \times 3 \div 3$

4.

1 + 4 — 4, × 4 ÷ 4
2 + 4 — 4, × 4 ÷ 4
3 + 4 — 4, × 4 ÷ 4
4 + 4 — 4, × 4 ÷ 4
5 + 4 — 4, × 4 ÷ 4
6 + 4 — 4, × 4 ÷ 4
7 + 4 — 4, × 4 ÷ 4
8 + 4 — 4, × 4 ÷ 4
9 + 4 — 4, × 4 ÷ 4
10 + 4 — 4, × 4 ÷ 4

5.

1 + 5 — 5, × 5 ÷ 5
2 + 5 — 5, × 5 ÷ 5
3 + 5 — 5, × 5 ÷ 5
4 + 5 — 5, × 5 ÷ 5
5 + 5 — 5, × 5 ÷ 5
6 + 5 — 5, × 5 ÷ 5
7 + 5 — 5, × 5 ÷ 5
8 + 5 — 5, × 5 ÷ 5
9 + 5 — 5, × 5 ÷ 5
10 + 5 — 5, × 5 ÷ 5

6.

1 + 6 — 6, × 6 ÷ 6
2 + 6 — 6, × 6 ÷ 6
3 + 6 — 6, × 6 ÷ 6
4 + 6 — 6, × 6 ÷ 6
5 + 6 — 6, × 6 ÷ 6
6 + 6 — 6, × 6 ÷ 6
7 + 6 — 6, × 6 ÷ 6
8 + 6 — 6, × 6 ÷ 6
9 + 6 — 6, × 6 ÷ 6
10 + 6 — 6, × 6 ÷ 6

7.

1 + 7 — 7, × 7 ÷ 7
2 + 7 — 7, × 7 ÷ 7
3 + 7 — 7, × 7 ÷ 7
4 + 7 — 7, × 7 ÷ 7
5 + 7 — 7, × 7 ÷ 7
6 + 7 — 7, × 7 ÷ 7
7 + 7 — 7, × 7 ÷ 7
8 + 7 — 7, × 7 ÷ 7
9 + 7 — 7, × 7 ÷ 7
10 + 7 — 7, × 7 ÷ 7

8.

1 + 8 — 8, × 8 ÷ 8
2 + 8 — 8, × 8 ÷ 8
3 + 8 — 8, × 8 ÷ 8
4 + 8 — 8, × 8 ÷ 8
5 + 8 — 8, × 8 ÷ 8
6 + 8 — 8, × 8 ÷ 8
7 + 8 — 8, × 8 ÷ 8
8 + 8 — 8, × 8 ÷ 8
9 + 8 — 8, × 8 ÷ 8
10 + 8 — 8, × 8 ÷ 8

9.

1 + 9 — 9, × 9 ÷ 9
2 + 9 — 9, × 9 ÷ 9
3 + 9 — 9, × 9 ÷ 9
4 + 9 — 9, × 9 ÷ 9
5 + 9 — 9, × 9 ÷ 9
6 + 9 — 9, × 9 ÷ 9
7 + 9 — 9, × 9 ÷ 9
8 + 9 — 9, × 9 ÷ 9
9 + 8 — 9, × 9 ÷ 9
10 + 9 — 9, × 9 ÷ 9

In reciting these tables the pupils should speak only the results, thus (table 8): 9, 1, 8, 1; 10, 2, 16, 2; 11, 3, 24, 3; 12, 4, 32, 4, etc. The tables may be recited in order, and promiscuously, the teacher pointing to the initial number. Promiscuous tables can also be arranged.

The comma (,) is inserted before the sign of multiplication (×) in the above tables to indicate that the preceding number is to be subtracted from the sum of the first two numbers, and that the *difference* is to be multiplied by the number after the sign ×. If the comma were omitted, the operations indicated by the signs × and ÷ would take precedence and the *quotient* would be subtracted from the sum of the first two numbers. See page 66.

FRACTIONS.

On page 121 of the Primary Arithmetic, it is suggested that pupils be taught, at this point, to divide a material unit into halves, fourths, etc., and to add and subtract the fractional parts of the unit, thus obtained. This instruction can best be given orally.

The illustration at the left represents a whole melon, a melon cut into two equal pieces, or halves, and a melon cut into four equal pieces, or fourths. Apples or pears can be used, and great pains should be taken to cut them into *equal* pieces, since a fraction is one or more of the *equal parts* of a unit.

By cutting two apples into halves, two into fourths, and two into eighths, pupils may be taught to add and subtract halves, fourths, eighths, etc.

Put together the corresponding fractional parts of the apple, in each case, and ask, How many halves are one half and one half? "Two halves." One half and two halves? "Three halves." Two halves and two halves? "Four halves."

How many fourths are one fourth and one fourth? "Two fourths." One fourth and two fourths? "Three fourths." Two fourths and two fourths? "Four fourths." Two fourths and three fourths? "Five fourths." Three fourths and three fourths? "Six fourths."

One fourth taken from two fourths leaves how many fourths? "One fourth." One fourth from three fourths? "Two fourths." One fourth from five fourths? "Four fourths." Two fourths from three fourths? "One fourth." Two fourths from five fourths? "Three fourths." Two fourths from seven fourths? "Five fourths."

In like manner teach the addition and subtraction of eighths.

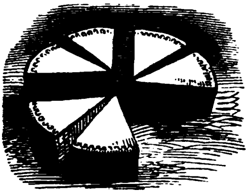
Next divide material units into thirds, sixths, etc., and teach the pupils to add and subtract thirds, sixths, etc., as above.

When pupils have a clear idea of halves, fourths, eighths, thirds, sixths, etc., and can add and subtract such fractional parts of a unit, they may then be taught to express these fractions by means of figures. Show that the figure below the line, in $\frac{3}{4}$, for example, denotes the number of equal parts into which the unit is divided, and that the number above the line denotes the number of these parts taken.

Write on the blackboard,

$$\frac{1}{4},\quad \frac{3}{4},\quad \frac{2}{4},\quad \frac{5}{4},\quad \frac{6}{4},\quad \frac{7}{4},$$
$$\frac{1}{8},\quad \frac{3}{8},\quad \frac{2}{8},\quad \frac{5}{8},\quad \frac{6}{8},\quad \frac{7}{8},$$
$$\frac{1}{3},\quad \frac{2}{3},\quad \frac{5}{3},\quad \frac{4}{3},\quad \frac{8}{3},\quad \frac{6}{3},$$
$$\frac{1}{6},\quad \frac{3}{6},\quad \frac{5}{6},\quad \frac{2}{6},\quad \frac{7}{6},\quad \frac{4}{6},$$

and, pointing successively to the different fractions, require the pupil to give the number of parts into which the unit is divided, and the number of these parts taken. At first, require the pupil to present the parts of the material unit corresponding to the fraction. If, for example, the fraction is $\frac{6}{8}$, have six of the eight equal parts of a material unit grouped together by the pupil.



In the absence of material units, the idea of a fraction and the combination of fractions may be very well shown by drawing equal circles on the blackboard and dividing them into equal parts to represent halves, fourths, eighths, thirds, sixths, etc.

DENOMINATE NUMBERS.

The more common denominations of currency, measure, and weight should be early taught to children. Such instruction should be a part of the general oral course in primary schools.

The first step in teaching these denominations is to develop the primary ideas involved, and the next is to impart a clear and definite conception of the value of the different units and their relations to each other. This can best be done by the use of their *representatives* in computing, measuring, weighing, etc., as is indicated below. Young pupils should not be required to memorize tables of denominate numbers before they have acquired a practical knowledge of their meaning and use.

United States Money.

The teacher should be supplied with the smaller pieces of money, coin and paper, in common use, including the *cent*, *two-cent*, *three-cent*, *five-cent*, *ten-cent*, *twenty-cent*, *quarter-dollar*, *half-dollar*, and *dollar*.

The first lesson may properly be devoted to the several coin pieces—the comparative value of the metals of which they are composed, the comparative value of the pieces, etc.

Hold up, successively, a cent, a two-cent piece, a three-cent piece, a five-cent piece (half-dime), and a ten-cent piece (dime), and ask the pupils to give you the value of each in cents. Then ask, How many cents in two two-cent pieces? "Four cents." In four two-cent pieces? In five?

How many cents in two three-cent pieces? "Six cents." In three? In five? In ten?

How many cents in two half-dimes? "Ten cents." In three half-dimes? In five? In eight? In ten?

How many cents in two dimes? "Twenty cents." In four dimes? In six? In seven dimes? In ten?

How many half-dimes in ten cents, in twenty cents, in twenty-five cents? In fifty cents? In one dollar?

How many cents in one dollar? How many cents in a half-dollar? In a quarter-dollar?

How many dimes in one dollar? In a half-dollar? In a quarter-dollar? "Two and a half dimes." In twenty cents? In forty cents? In sixty cents?

How many quarters in a half-dollar? In seventy-five cents? In two dollars?

Next give the pupils practice in buying and selling, "making change," etc. Put several pieces of money into the drawer of a table or desk, and let one of the pupils act the part of a shopkeeper. Give the other pupils, successively, pieces of money, and let them buy articles, or books, slates, pencils, etc., and pay for them, securing the right change. This exercise may be made an interesting amusement for recess.

Write the following table on the blackboard, and have the pupils copy and commit it to memory:

TABLE.

10 mills	(*m.*) . .	are **1** cent . .	*c.* or *ct.*
10 cents	. . . .	are **1** dime . .	*d.*
10 dimes	. . . .	are **1** dollar . .	**$.**

$1 = 10*d.* = 100*c.* = 1000*m.*

Long Measure.

The teacher should be supplied with a foot-rule, divided into inches, a yard-stick, and a piece of tape or cord at least one rod in length. Each pupil should have a six-inch rule. A narrow strip of strong paper, accurately divided into inches, will answer.

Develop the idea of length or distance. Hold up two objects of nearly equal length and let the pupils "guess" which is the longer. Test their guess by putting the objects together.

Draw a line on the blackboard one inch in length, another six inches in length, and divide it accurately into inches.

Draw other lines on the blackboard of different lengths, less than a foot, and ask the pupils to guess the length of each in inches. Test the accuracy of each guess by applying the rule.

Draw lines in different positions,—vertical, horizontal, oblique—and have their lengths guessed and the lines measured.

Ask the pupils to draw a line on their slates, three inches long, four inches long, six inches long, two inches long, etc., and test the accuracy of each by applying the six-inch rule.

Have the length of objects, as books, slates, pen-holders, hats, etc., guessed and then measured.

Draw a line one foot in length on the blackboard, and divide it into inches. Draw other lines, from one to three feet in length, in different positions, and have their lengths guessed and measured. Have the pupils guess and then measure the length and width of desks, doors, windows, blackboard, etc.

Show the pupils a yard-stick, and have the length and width of the room measured, the length and width of the school-yard, etc. Let them measure from a ball of yarn or cord or tape, five yards; eight yards, etc.

Supply the pupils with a piece of tape or cord one rod in length, and let them measure it with the yard-stick, and thus learn that its length is just 5½ yards.

With this rod-measure, let them determine the width of the street in rods; the distance between objects two and two, etc. Let them place two objects five rods apart; ten rods apart; eight rods apart, etc., and test the accuracy of their guess by measurement.

Select a well known object one mile or half a mile from the school-house, and have the pupils compare this distance with the distance to other objects. Teach the pupils that a mile contains 320 rods.

Write this table on the blackboard for the pupils to copy and commit to memory:

TABLE.

12	inches (*in.*) .	are **1** foot	*ft.*
3	feet . . .	are **1** yard	*yd.*
5½	yards . . .	are **1** rod	*rd.*
320	rods . . .	are **1** mile	*mi.*

1 mi. = **320** rd. = **5280** ft.

It is not deemed advisable to teach the tables of square measure and cubic measure to primary pupils, but it may be well to give them a clear idea of a square inch, a square foot, and a square yard; also of a cubic inch, a cubic foot, and a cubic yard. See Intermediate Arithmetic, articles 89 and 92.

Dry Measure.

The teacher should be supplied with a pint-measure, a quart-measure, a peck-measure, a bushel-measure, several paper sacks of different sizes, and a bushel of some kind of grain or fruit. A half-peck measure and a half-bushel may take the place of the sacks.

Teach the names of the several measures, and make the pupils familiar with them by measuring out two pints of grain, three pints, five pints, etc.; two quarts, four quarts, three quarts, six quarts, etc.; two pecks, three pecks, and four pecks.

Next, let the pupils determine how many pints are equal to one quart; how many quarts are equal to one peck; and how many pecks are equal to one bushel. This may be done, in each instance, by both filling and emptying the larger measure, by using the smaller.

When the pupils have thus learned the comparative capacities of the different measures, let them ascertain how many quarts of grain will fill a common pail or bucket, the sacks of different sizes, boxes, baskets, etc.

Ask, How many pints in a quart? In two quarts? In five quarts? In ten quarts?

How many quarts in a peck? In three pecks? In a bushel? In a half-bushel? In a half-peck?

How many pints in a peck? In a half-peck? In a bushel? In a half-bushel?

How many quarts in two pints? In four pints? In eight pints? In twelve pints? In ten pints?

How many pecks in eight quarts? In sixteen quarts? In thirty-two quarts? In forty quarts?

Have the pupils copy and memorize this

TABLE.

2 pints (*pt.*) . . are 1 quart . . . . *qt.*
8 quarts . . . . are 1 peck . . . . *pk.*
4 pecks . . . . are 1 bushel . . . . *bu.*

1 bu. = 4 pk. = 32 qt. = 64 pt.

Liquid Measure.

The teacher should be supplied with a pint-cup, a quart-cup, a gallon measure, and a bucket of water.

Teach the names of the three measures and their comparative capacities, in the manner above described.

Show the pupils that a quart of water is less than a quart of grain; compare the two quart measures.

Let the pupils ascertain how many quarts of water will fill a common pail or bucket; how much will fill a common tumbler, etc.

How many pints in two quarts? In three quarts? In four quarts or a gallon?

How many quarts in a gallon? In three gallons? In five gallons? In six gallons? In ten gallons?

How many gallons in eight quarts? In twelve quarts? In ten quarts? In twenty quarts?

Have the pupils copy and memorize this

TABLE.

4 gills (*gi.*) . . . are 1 pint . . . . *pt.*
2 pints . . . . are 1 quart . . . *qt.*
4 quarts . . . . are 1 gallon . . . *gal.*

1 gal. = 4 qt. = 8 pt. = 32 gi.

Avoirdupois Weight.

The teacher should be supplied with a pair of grocer's scales, and a balance, with the usual weights; also parcels of different substances, as sand, oats, sawdust, etc., each weighing just one pound.

Let a pupil take the pound weight in one hand and the parcel of sand in the other, and tell which seems the heavier. Let another lift the weight and the parcel of oats, and tell which seems the heavier. Let another lift the weigh and the parcel of sawdust. Put the pound weight in one end of the balance and the parcels, successively, in the other, and thus show that each weighs just a pound.

Take the ounce weight and show that it would take sixteen such weights to balance the pound weight. Ask, How many ounces in a pound? "Sixteen." Hold up, successively, the ounce-weight, two-ounce weight, three-ounce weight, four-ounce weight, etc., and have the pupils give the number of ounces in each?

How many eight-ounce weights equal one pound? "Two." How many four-ounce weights?

Let the pupils weigh several parcels by means of the balance; also articles by means of the scales.

Have the pupils guess the weight of a gallon of water, a peck of oats, a quart of sand, a brick, etc., and test their accuracy by weighing each article.

Let them guess the weight of one another; the weight of a bushel of wheat; a bushel of oats; a barrel of flour, etc. Give them some idea of a ton.

Have them copy and memorize this

TABLE.

16	drams (*dr.*) .	are	**1**	ounce	*oz.*
16	ounces . . .	are	**1**	pound . . .	*lb.*
100	pounds . . .	are	**1**	hundred-weight	*cwt.*
20	hundred-weights	are	**1**	ton	*T.*

Time Measure.

When did to-day begin? "At 12 o'clock last night." When will to-day end? "At 12 o'clock to-night." What is 12 o'clock at night called? "Midnight." How long, then, is a day? "From midnight to midnight."

A day is divided into twenty-four hours. How many hours from midnight to noon? "Twelve." From noon to midnight? "Twelve." How often does the clock strike? "Once an hour." If you should study from the striking of the clock to its next striking, how long would you study? "One hour." How many times does the clock strike in a day? "Twenty-four times."

An hour is divided into sixty minutes. Let the school be silent *one minute*. How many times this silence would be an hour? "Sixty times." How many minutes long is our usual recess? How many minutes have you been reciting?

Put your finger on your pulse and count silently sixty pulsations. It has taken a little less than a minute. Some clocks tick once a *second*, or sixty times a minute. How many seconds in a minute? "Sixty."

How many days in a week? "Seven days." What is the name of the first day of the week? "Sunday." The second day? The third day? The fourth day? The fifth day? The sixth day? The seventh day? Name the days of the week in their order. What day is before Thursday? What day is after Thursday?

How many days in a year? Explain to the pupils what is meant by leap-year. How many days in leap-year? How often does leap-year occur?

How many months in a year? What is the first month? "January." What is the second month? Name all the months in their order?

Into how many seasons is the year divided? "Four seasons." Name them in their order.

What months constitute Winter? Spring? Summer? Autumn, or Fall?

Have the pupils copy and memorize the following tables:

TIME TABLE.

60	seconds (*sec.*)	are	**1**	minute	*min.*
60	minutes	are	**1**	hour	*h.*
24	hours	are	**1**	day	*d.*
365	days	are	**1**	common year	*c. yr.*
366	days	are	**1**	leap-year	*l. yr.*
100	years (365¼ d.)	are	**1**	century	*C.*
7	days	are	**1**	week	*w.*
12	months	are	**1**	year	*yr.*

MISCELLANEOUS TABLE.

24	sheets of paper	are	**1**	quire.
20	quires of paper	are	**1**	ream.
12	things	are	**1**	dozen.
20	things	are	**1**	score.
196	pounds of flour	are	**1**	barrel.

The foregoing seven tables are in common use, and they should have a place in the primary course. Several of the tables included in most elementary arithmetics are now obsolete, and several others are little used.

INTERMEDIATE ARITHMETIC.

General Suggestions.

This book is designed to follow the more elementary course embodied in the Primary Arithmetic, and the best results will be attained by first using the Primary.

When the Intermediate is the first book used by the pupil, it should be preceded by an oral course equivalent to the first thirty lessons in the Primary, and then the successive lessons should be accompanied by the corresponding lessons in the Primary from the thirtieth to the ninetieth. In most cases it will be found best to preface the lessons in sections II, III, IV, and V of the Intermediate, by a review of the corresponding lessons in the Primary,—the "tables" being carefully developed and memorized, and the "written exercises" used as slate and blackboard lessons, with additional blackboard drills. See previous pages.

The two books may thus be successfully united, but it will be found true economy in time and a gain in results to begin with the Primary Arithmetic. The preparatory oral instruction in numbers, given in most schools, is too abstract and monotonous. It needs to be supplemented by the more varied and systematic exercises of a first book.

The fundamental operations are, for convenience, treated separately, with added review lessons combining them. The book lessons should be supplemented by oral and blackboard drills, and in them the several processes may be combined to any extent that may seem desirable. Models of these combinations are given on pages 26 and 39.

Mental Exercises.

The solutions of the mental problems should be concise and simple. Pupils, at this stage of their advancement, are not helped by a minute and formal statement of every condition involved in a problem, and, at no stage, is the reasoning faculty trained by the repetition of what has been aptly called "logical verbiage." For numerous "Models of Analysis," see Part II, pages 3 to 10.

Written Exercises.

When the mental problems have been solved orally, they should next be solved on slate or paper, as a preparation for the written problems. This will not only make the induction of the written process from the oral analysis an easy step, but it will also increase the number of written solutions. See page 12.

All the problems in an assigned lesson should be solved by the pupils on slate or paper, and *the solutions should be brought to the recitation for the teacher's inspection and criticism.* These solutions should be made in an *approved form*, and they should be arranged in a neat and systematic manner. A little instruction will enable pupils to make an economic use of space in slate and blackboard work, and, at the same time, to arrange each solution in an intelligible form. The accuracy and neatness of each pupil's work should receive the attention of the teacher.

Definitions and Principles.

The definitions and principles should be deduced and stated by the pupils under the guidance of the teacher, and this can usually be done in connection with the solution of the problems. For illustrations, see page 58.

Rules.

The rules should be deduced from the processes, and stated by the pupils. See page 12. The true order is this:

1. A mastery of the process without reference to the rule.
2. The recognition and statement of the successive steps in order.
3. The combination of these several statements into a general statement.
4. A comparison of the rule thus formed with the author's rule.
5. The memorizing of the approved rule.

For illustrations of the method, see pages 59 and 60.

Summary of General Method.

The above suggestions combined give this order for teaching each operation:

1. The oral drill on the mental exercises.
2. The induction of the written process from the oral solutions, under the guidance of the teacher.
3. The solving of problems, selected from the mental exercises, on slate or paper.
4. The solving of the written problems on slate or paper.
5. The induction of the rule from the written process, and the memorizing of an approved rule.
6. The induction and memorizing of definitions and principles.

The step last stated may sometimes be taken in connection with the first and third, and the rule may be deduced and stated as soon as the pupils are sufficiently familiar with the process. See page 12.

NOTATION AND NUMERATION.

The Decimal System.

The first lesson is devoted to the reading and writing of numbers expressed by one, two, and three figures. It is but a concise statement of what is already familiar to pupils who have taken the primary course.

When the Primary Arithmetic has not been mastered, this lesson should be prefaced by the drills sketched on pages 22–24 and 27–32, supplemented with necessary *practice*. It is unwise to attempt to teach the formal statement of the principles of notation before the pupil is able to read and write any number expressed by three figures, and these principles can be better taught still later in the course.

If the units' period has been thoroughly mastered, the pupils will find very little difficulty in the second lesson. *The important thing is to make each step familiar by practice.* Teachers often make notation difficult by *too much* explanation. The fanciful illustrations sometimes given are more difficult to understand than the subject itself. What is needed is *progressive practice.*

Definitions and Principles.

When pupils can readily read and write any number composed of one to six figures, they can be easily taught the principles of the decimal system. Most of the difficulty in understanding these principles is due to the fact that the pupil is not *prepared* for their study. The first step is the mastery of the *art* of writing numbers. To begin with the definitions and principles of the decimal notation and end with practice, is to reverse the natural order of teaching the subject to children.

In teaching beginners the division of the orders into periods, it may be well to call the first order in each period *ones*, the term units being used as the name of the first period. This may avoid confusion in young minds. The names of the three orders of each period will be *ones*, *tens*, *hundreds*, with the name of the period properly added. The name of the first period is usually omitted in reading the orders separately.

There is no advantage in teaching pupils at this stage the names of more than five periods.

Roman Notation.

The first instruction in the Roman method of notation should be given in connection with the reading lessons, and other lessons thus numbered. The method is largely mastered when the child can read and write any number from I to C, and this should be accomplished before the Intermediate Arithmetic is reached.

Many teachers do not give their pupils practice in the *writing* of numbers by the Roman method, and hence they make slow progress in reading numbers thus expressed. Pupils should be drilled both in reading and writing numbers by letters, and the two processes should be taught together. When this is done, each aids the other.

ADDITION.

Oral and Written Exercises.

The progressive order of the lessons is designed to afford the pupil the necessary practice in combining the lower digital numbers, 1, 2, 3, etc., before the higher are reached. Follow this order, and see that each lesson is thoroughly mastered before the succeeding one is undertaken.

Observe the careful grading of the Written Exercises, and see that each step is made familiar before the next is taken.

Review the Tables and Blackboard Drills before each lesson in the book, and give in connection the corresponding lessons in the Primary Arithmetic. See pages 24 and 25. If the Primary has not been previously studied, combine the corresponding lessons of the two books, as suggested on page 53.

Multiply the drills in adding by 2's, 3's, 4's, etc., until the requisite accuracy and rapidity are secured.

Definitions.

The definition of addition may be taught by writing, say, 2, 4, and 3 on the blackboard, adding them, and then asking, What have we done? "We have added 2, 4, and 3." What number have we obtained? "9." What is this number called? "The *sum* or *amount*." [These terms have been used in connection with the problems.]

$$\begin{array}{r} 3 \\ 4 \\ 2 \\ \hline 9 \end{array}$$

How many ones or units in 2 and 4 together? "6." In 2, 4, and 3 together? "9." Write 3, 5, and 6 on the blackboard, and ask, How many units in 3 and 5? "8." In 3, 5, and 6? "14." How many units in the sum of any two or more numbers? "The sum of two or more numbers contains as many units as all the numbers together."

The addition of numbers is called a *process*. What kind of a process is addition? "*Addition is the process of finding the sum of two or more numbers.*" What is meant by the sum of two or more numbers? "*The sum is a number that contains as many units as all the numbers added.*"

If preferred, these two definitions may be united, giving this definition: "*Addition is the process of finding a number that contains as many units as several given numbers.*" It is, however, believed to be better to define addition by the use of the known term *sum* (or amount), and then to define the term sum. The pupil is already familiar with the process of adding and with the term sum, and the definitions are but formal statements of what is already known.

Rule.

The rule for addition may be deduced from the process thus: What is the first step in solving a written problem in addition? "Write the numbers so that figures of the same order shall stand under each other." Write what numbers? "Write the numbers *to be added.*" Is it proper to say "*figures* of the same order?" "It is not." What is proper? "Figures denoting units of the same order." Can two or more numbers be written under each other? [Show that they can not be thus written.] How, then, must the numbers of the same order be written? "In the same column." When the numbers are written, what do you do? "Draw a line underneath."

What then is the first step in addition?

"*Write the numbers to be added so that the figures denoting units of the same order shall stand in the same column, and draw a line underneath.*"

Let this be repeated by different pupils until it can be given without hesitation.

Show that only numbers of the same order can be added. Illustrate by taking groups of different objects, as 3 blocks and 4 sticks, 2 dimes and 3 cents, etc. Develop and state the general principle that *only like numbers can be added.*

What is the second step? "Add each column, and write the sum underneath." With which column do you begin? "With the right-hand or units' column." [Show why this is done.] When can you write the sum of the numbers in a column underneath? "When the sum is less than ten." Express all this in one sentence.

"*Begin with units and add each column, writing the sum, when less than ten, underneath.*"

What is done when the sum exceeds nine? "When the sum of any column exceeds nine, write the right-hand figure under the column added, and add the left-hand figure to the next column." Can we add a "*figure* to the next column?" "We can not." What can we do? "We can add the *number* denoted by the left-hand figure." Suppose the sum of a column should be 125, what number would be added to the next column? "12." How many figures express 12? "Two." What, then, is added to the next column? "The number denoted by the left-hand figure or *figures*." Now repeat your statement.

"*When the sum of any column exceeds nine, write the right-hand figure of the sum underneath the column added, and add the number denoted by the left-hand figure or figures with the next column.*"

Let this be repeated by different pupils until it can be given without hesitation.

How is the sum of the left-hand column written? "The entire sum is written, the right-hand figure being written underneath the column." How may this be expressed?

"*Write the entire sum of the left-hand column.*"

How may you test the correctness of the result?

"*To test the correctness of the result, add the columns downward.*"

When the rule has thus been developed, have the several steps repeated in connection with the solution of several problems, one pupil giving the first step, another the second, and so on. Continue the drill until the pupils can repeat the entire rule with accuracy and without hesitation.

The rule may thus be taught in connection with the first or any succeeding lesson, as may seem best.

SUBTRACTION.

Introduce each lesson by a review of the Table and Blackboard Drills, given in connection with the corresponding lessons in the Primary Arithmetic.

Observe the progressive order of the exercises. In the written exercises of the first two lessons, no term of the subtrahend exceeds the corresponding term of the minuend.

In the third lesson one or more of the terms of the subtrahend exceed the corresponding terms of the minuend. How is this difficulty best mastered? There are three methods:

1. Ten may be added to the term of the minuend and one to the next higher term of the subtrahend, thus increasing both minuend and subtrahend *equally*.

2. One may be mentally taken from the next higher term of the minuend, and the ten, obtained by reducing this one to the next lower order, may be added to the proper term of the minuend. In subtracting the next term of the subtrahend, the term above it is considered diminished by one.

When the next higher term of the minuend is 0, one is taken from the next higher term whose value is one or more, and the necessary reductions to lower orders are made. In subtracting the terms of the subtrahend, each 0 of the minuend above is considered 9.

3. Instead of considering the next higher term of the minuend diminished by one, as in the second method above, one may be added to the next higher term of the subtrahend. It may be easily shown that adding one to the term of the subtrahend is *in effect* the same as subtracting one from the term of the minuend.

The first of the above methods is the most easily explained. It is easy to show that adding ten to a term of the minuend and one to the next higher term of the subtrahend increases both minuend and subtrahend *equally*, and hence does not affect the difference. That increasing both minuend and subtrahend equally does not affect their difference, may be shown by taking two numbers as 7 and 4, and adding say 2 to each. The difference between 7 and 4 is 3, and the difference between 9 and 6 is 3.

The second method is not easily explained when the next higher term or terms of the minuend are 0. It is difficult for a young pupil to see clearly the effect of the several changes on the value of the minuend.

Whatever the method employed, the term "borrow" should not be used in describing or explaining the process. In no one of the methods is there any borrowing.

The definitions and rule should be taught inductively. See pages 58–60. The first two parts of the rule may be deduced and stated in connection with the first two lessons. See page 79 for additional problems combining addition and subtraction.

MULTIPLICATION.

Review the Tables and Blackboard Drills in Primary Arithmetic, as in addition and subtraction.

Begin with the lower digital numbers and gradually introduce the higher. Continue the drills until the pupils can give the products with accuracy and rapidity. The pupil should never be under the necessity of referring to the multiplication table.

The written exercises are so graded that they introduce but one difficulty at a time, and each step is made familiar *by practice.*

In the first lesson the multiplier contains only one figure, and no one of the successive products exceeds nine. There is no "carrying" in this lesson.

In the second lesson the multiplier contains one figure, and one or more of the products exceed nine, thus involving the process of carrying.

In the third lesson the multiplier contains two or more figures, and the process of forming and adding the partial products is mastered.

In the fourth lesson the multiplier contains one or more ciphers.

In the fifth lesson the multiplier or multiplicand ends with ciphers, and in the sixth lesson both factors end with ciphers.

In giving additional problems the teacher should observe this progressive order.

The definition of multiplication is deduced in the "Suggestions to Teachers," Intermediate Arithmetic, page (v). The other definitions, and also the rule, should be taught inductively.

DIVISION.

Preface each lesson by a review of the Table and Blackboard Drills, as in the foregoing sections. Introduce the digital numbers successively, beginning with the lower, and teach division as the converse of multiplication.

Short Division.

The first two lessons are devoted to short division. In the first, each successive term of the dividend *is exactly divisible by the divisor*, and hence there is no remainder, and no consequent reduction to form the next partial dividend. When this step is mastered, the next is easily taken, and the process of short division is but little more difficult than the mental exercises.

It is the practice of some teachers to introduce long division before short division. There seems to be no special advantage in this in practice, and it is certainly not the natural order. Short division is nearly, if not quite, as easily mastered before as after long division, and its mastery assists the pupil in learning the more difficult process of long division.

Long Division.

The principal difficulty in long division is in determining the successive figures of the quotient. This difficulty may be greatly lessened by the following method:

1. Begin with divisors that contain but one figure. The examples in the second lesson, previously solved by short division, may be used for this step.

2. Next take examples in which each successive quotient figure is found *by dividing the left-hand term of each successive partial dividend by the left-hand term of the divisor.*

3. Then take examples in which the two left-hand figures of the divisor are 1 and 0 respectively. The number denoted by these two figures will be the trial divisor.

The following solutions will illustrate these three steps:

(1.)	(2.)	(3.)
7)4704(672	224)4704(21	1036)452732(437
42	448	4144
50	224	3833
49	224	3108
14		7252
14		7252

The first four problems below illustrate the second step, and the next four illustrate the third step:

1. Divide 9734 by 314. *Ans.* 31.
2. Divide 49203 by 213. *Ans.* 231.
3. Divide 99584 by 3112. *Ans.* 32.
4. Divide 888408 by 216. *Ans.* 4113.

5. Divide 662528 by 1024.	*Ans.* 647.
6. Divide 3531648 by 1088.	*Ans.* 3246.
7. Divide 4851122 by 1069.	*Ans.* 4538.
8. Divide 4958961 by 10267.	*Ans.* 483.

When pupils can solve readily examples similar to the above, they will find comparatively little difficulty in the third and succeeding lessons.

Review Problems.

Additional problems for a review of the four operations are given on page 79. These problems are so graded that they may be used to supplement those combining addition and subtraction, and those combining addition, subtraction, and multiplication.

DRILLS IN RAPID COMBINATION.

The lessons may be accompanied by oral and blackboard drills in the rapid combination of numbers by addition, subtraction, multiplication, and division. The rapidity secured by these drills has won for them the appellation of "Lightning Combinations." The following are illustrations of the oral drills:

1. Take 5, add 7, subtract 4, multiply by 7, add 4, divide by 6, subtract 3, multiply by 9, add 5, add 4, divide by 8: what is the result?

2. Take 9, multiply by 6, add 7, subtract 5, divide by 8, add 5, multiply by 6, add 9, divide by 9, multiply by 5, add 4, divide by 7, multiply by 8: result?

3. Take 65, subtract 9, divide by 8, multiply by 6, add 9, subtract 6, divide by 9, add 4, multiply by 8, add 9, divide by 9, divide by 3: result?

4. Take 15, add 12, add 9, subtract 4, divide by 8, add 4, multiply by 9, subtract 8, divide by 8, multiply by 7, add 9, add 7, divide by 8, divide by 3: result?

At first the class may give orally each result, but, as soon as the process is familiar, only the final result should be given. The teacher should dictate slowly at first, increasing in rapidity as skill is acquired by the class.

The drills may be varied by writing the exercises on the blackboard, and pointing successively to the combinations to be made. The following are illustrations, the operations being performed in the order indicated:

1. $5 + 7, -4, \times 7, +4, \div 6, -3, \times 9, +5, +4, \div 8, =$ what?

2. $9 \times 6, +7, -5, \div 8, +5, \times 6, +9, \div 9, \times 5, +4, \div 7, \times 8, =$ what?

3. $65 - 9, \div 8, \times 6, +9, -6, \div 9, +4, \times 8, +9, \div 9, \div 3, =$ what?

4. $15 + 12, +9, -4, \div 8, +4, \times 9, -8, \div 8, \times 7, +9, +7, \div 8, \div 3, =$ what?

5. $8 \times 9, \div 12, \times 7, +7, \div 7, +3, \times 9, -6, \div 7, \div 6, =$ what?

6. $56 \div 7, \times 9, -5, -4, \div 7, \times 9, +3, \div 12, \times 6, +6, \div 8, \div 6, =$ what?

7. $42 + 6, +9, -3, \div 9, \times 8, +9, +7, \div 8, \times 10, +1, \div 9, \div 3, =$ what?

8. $6 + 5, \times 7, +4, \div 9, \times 7, -5, +6, \div 8, \times 3, +9, +7, \div 10, \times 12, =$ what?

9. $47 + 9, \div 7, \times 5, +8, -3, \div 9, \times 12, +3, \div 7, \times 8, \div 9, \times 4, =$ what?

10. $100 - 25, +6, \div 9, \times 6, +10, \div 8, \times 7, +7, \div 9, \times 7, -1, \div 8, =$ what?

NOTE.—The comma (,) is used in the above exercises to indicate that the operations expressed by the signs +, —, ×, and ÷, are to be performed *in the order in which they occur.*

If the comma were not used, the operations indicated by × and ÷ would take precedence over the + or — immediately before. The operations in the first exercise, if written without the comma, would be performed in this order: $5 + 7 - (4 \times 7) + (4 \div 6) - (3 \times 9) + 5 + (4 \div 8)$. The sign + or — would include all that occurs *between it and the next sign of + or —*.

In writing the above exercises, it is only necessary to place a comma before the sign × or ÷ *when following a number that is preceded by the sign + or —*. But to avoid all ambiguity and mistakes, it is best to place the comma before each sign after the first.

COMMON FRACTIONS.

General Suggestions.

These lessons are designed to impart an *elementary knowledge* of fractions, and hence only the more elementary principles and operations are presented, and each in the simplest manner.

But *one* method of performing an operation is given, and all illustrations and processes are based on the idea that *a fraction is one or more of the the equal parts of a unit.* The fact that a fraction may be considered an "unexecuted division" is presented in the Complete Arithmetic where it may be taught without confusing the pupil. The theory and use of "cancellation," as an abbreviated process, is deferred until the process itself is mastered.

Not only the idea of a fraction, but each principle and process should be developed by means of visible objects. The manner in which this may be done is clearly shown in the oral and illustrative exercises in the book and also in this MANUAL, page 41.

The divisions and subdivisions of the unit may be easily shown on the blackboard by using lines, circles, and other figures as objects. This use of objects should not be carried too far. It is but the first step in the lesson. The principles, given in connection with the rules, should be thoroughly taught.

Great care should be taken to make prominent the fact that the parts of the unit must be EQUAL. It should also be made clear that a fraction is not an *expression* of one or more of the equal parts of a unit. The fraction is one thing, and its expression is quite another thing. A fraction may be expressed by words or figures, but neither the words nor the figures are the fraction. They are only its *sign* or *expression.*

It is a similar error to call a figure a number. The figure 2 is a character that represents the number two. Numbers may be added, but figures can not be added.

Pupils should be taught to reduce fractions to a common denominator *by inspection.* The process of finding the least common multiple of the denominators should only be used when the common denominator can not otherwise be determined.

The method of dividing one fraction by another by first reducing them to a common denominator is easily understood by young pupils, and, when mastered, it affords an easy explanation of the method which inverts the terms of the divisor. See Complete Arithmetic, page 64. Since complex fractions are never met with in practical life, they are not treated in this elementary book. They are fully treated in the Complete Arithmetic.

See page 80 for drill problems to supplement each lesson in fractions.

DECIMAL FRACTIONS.

The Idea of a Decimal Fraction.

If pupils have a clear idea of a fraction, it is unnecessary to divide a material unit into tenths, hundredths, etc., to show the nature of a decimal fraction. This may be easily accomplished by asking such questions as the following:

If a unit be divided into ten equal parts, what is one part called? "One tenth." If a tenth be divided into ten equal parts, what is one part called? "One hundredth." If a hundredth be divided into ten equal parts, what is one part called? "One thousandth."

Write on the blackboard $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$; $\frac{5}{10}$, $\frac{5}{100}$, $\frac{5}{1000}$, etc., and by questions lead the pupils to see their relative value. Show that the division of a unit into tenths, hundredths, thousandths, etc., is called a *decimal* division, since the divisor is ten (*decem*), and that the resulting fractions are called *decimal fractions*. Write on the blackboard $\frac{5}{10}$, $\frac{15}{100}$, $\frac{225}{1000}$, etc., and have the pupils show what each denominator expresses; also each numerator.

Do not confuse the pupil by teaching the error that these are not decimal fractions, because they are not expressed in the decimal form. The decimal fraction is one thing and its expression by figures is quite another thing. The fundamental idea of the decimal fraction is the *decimal division* of the unit, and any fraction whose denominator is a power of ten is a decimal fraction, *whatever be the mode by which it is expressed.* The nature of the fraction does not depend on the mode of its expression.

The attempt to separate fractions into two distinct and separate classes, called common and decimal, has resulted in much confusion. This classification can only be applied to fractions *expressed by figures.* It may be agreed to call every fraction expressed by two terms, as $\frac{3}{7}$, $\frac{3}{10}$, etc., a common fraction, and every fraction expressed in what is called the decimal form, as .3, .25, etc., a decimal fraction, and thus fractions expressed by figures may be divided into two separate classes. But this classification does not apply to fractions *expressed by words*, and every fraction may be thus expressed. Nor will it answer to call all fractions expressed by words common fractions, since this ignores the very basis of the classification. It is true that all fractions may be expressed in the common form, but it is also true that a class of fractions may be expressed in the decimal form. What kind of a fraction is *three tenths?* It may be expressed by figures in both forms, as $\frac{3}{10}$ and .3, and hence, *when expressed by words*, it belongs to both classes of fractions, or to neither class.

The limiting of the term decimal to fractions expressed by figures in the decimal form, makes it impossible to read or dictate a decimal fraction, since the expression of such a fraction in words would destroy its decimal character. The teacher can not say to the pupil, "Write in figures the decimal forty-five thousandths," for, according to the definition, forty-five thousandths is not a decimal until it is expressed in figures in the decimal form.

Such a limitation of the term decimal is an error. The fraction forty-five thousandths is as clearly a decimal when spoken by the teacher as it is when expressed in figures by the pupil.

It may be added that all arithmetics teach both the reading and the writing of decimals, and hence they all recognize the fact that decimal fractions may be expressed both by words and by figures. Most of them give an erroneous definition, and hence the teacher should take special pains to give the pupil a clear idea of the true nature of this class of fractions.

Notation and Numeration.

When the pupil has a clear idea of a decimal fraction, he may next be taught that such a fraction may be expressed by figures in two forms. Five-tenths may be written $\frac{5}{10}$, or .5, and seventeen hundredths may be written $\frac{17}{100}$, or .17. The first form ($\frac{5}{10}$ and $\frac{17}{100}$) is the common form, and the second (.5 and .17) is the decimal form.

Explain this decimal form. Since tenths, hundredths, thousandths, etc., arise from a decimal division, they decrease in value *tenfold*, and hence may be expressed on the decimal scale by writing tenths in the first order at the right of units, hundredths in the second order, thousandths in the third order, etc., and placing a period at the left of the tenths to show that the orders are fractional. Thus .3 denotes 3 tenths; .34 denotes 3 tenths and 4 hundredths, or 34 hundredths; and .345 denotes 3 tenths, 4 hundredths, and 5 thousandths, or 345 thousandths.

The pupil should next be drilled in reading and writing decimal fractions expressed by one, two, and three orders, until the requisite skill is acquired. If a pupil can not read or write any such decimal instantly, it is useless to attempt to teach him to read and write decimals composed of a greater number of orders. Whatever may be the method employed for teaching the notation of decimals, the one essential condition of success is *progressive practice.* The first step is the reading and writing of tenths; the second, the reading and writing of hundredths; the third, the reading and writing of thousandths, etc., and the pupil must be kept on each step until he is made familiar with it. No pupil can instantly read a decimal if he can not, at a glance, recognize the name of the last order at the right. To secure this result, *the practice of young pupils should be confined to the reading and writing of decimals composed of not more than six orders.*

Reduction of Decimals.

The third case in the reduction of decimals presents the only special difficulty, and this may be largely removed by a thorough knowledge of the first case. The pupil must see that each cipher annexed to the numerator in the process of division is a *decimal cipher*, and hence gives one decimal place in the quotient. Thus, $\frac{3}{4}$ is $3.00 \div 4$, which is .75; $\frac{3}{40}$ is $3.000 \div 40$, which is .075; $\frac{3}{400}$ is $3.0000 \div 400$, which is .0075. The fact that the denominator ends with ciphers increases the pupil's liability to make an error. To secure absolute accuracy require the resulting decimals to be changed back to common fractions in their lowest terms. Thus, $.75 = \frac{75}{100} = \frac{3}{4}$; $.075 = \frac{75}{1000} = \frac{3}{40}$; $.0075 = \frac{75}{10000} = \frac{3}{400}$.

Decimal Operations.

Since the value of the decimal orders increases tenfold from right to left, the same as the orders of integers, the processes of addition and subtraction present no difficulty.

If the pupil finds difficulty in determining the number of decimal places in the product in multiplication, let him supply the denominators and ascertain what their product will be. Thus, $.435 \times .65 = \frac{435}{1000} \times \frac{65}{100} = \frac{28275}{100000} = .28275$. If the oral exercises are thoroughly taught, most pupils will clearly see the principle that determines the number of decimal places in the product. When the principle is understood, the process will not be difficult. A mastery of the principle should precede the memorizing of the rule.

The problems in the division of decimals are so graded that the three cases are successively mastered. In the first step the dividend and divisor contain the same number of decimal places, and the quotient is an integer. In the second step the dividend contains more decimal places than the divisor, and the quotient contains one or more decimal places. In the third step the dividend contains less decimal places than the divisor and decimal ciphers are annexed to the dividend to supply the deficiency. If these three steps are carefully taken, the difficulties of the subject will be easily overcome.

To secure absolute accuracy in pointing require the pupil to prove his result by multiplying the quotient by the divisor. The three principles underlying the process of division, should be thoroughly taught.

See page 85 for additional problems for dictation.

UNITED STATES MONEY.

The lessons in the arithmetic should be introduced by oral lessons, designed to make pupils familiar with the value and use of the more common pieces of money, including coins, treasury notes, and bank notes. This instruction is sketched on page 44 as a part of the primary course. If previously given in that grade, it will be well to review it, thus making the pupils' knowledge fresh, and greatly adding to their interest in the exercises in the book.

The operations in United States money present no difficulty to pupils who have a knowledge of decimal fractions. When United States money is taught before decimals, the pupils should be made familiar with the first three decimal orders—tenths, hundredths, and thousandths. This can be done in one or two lessons by the method sketched on page 69.

When pupils can read any decimal composed of one, two, or three orders, and have learned that cents are *hundredths* of a dollar, and mills *thousandths*, they will readily understand why cents occupy the first two decimal orders and mills the third. The separatrix between dollars and cents corresponds with the decimal point.

The reduction of United States money may be taught as an application of the reduction of decimals, or as the reduction of denominate numbers. The exercises should be multiplied until the reduction of dollars to cents, dollars to mills, cents to mills, mills to cents, cents to dollars, etc., can be made instantly. Many classes have failed when subjected to an examination on these reductions.

Another process deserving special attention, is the division of one sum of money by another (Lesson IV). The first step is to reduce both dividend and divisor to *the same denomination.* Show that dollars can not be divided by cents, or cents by mills; that dollars must be divided by dollars, cents by cents, and mills by mills.

The lesson on Bills is one of great practical value, and should be thoroughly taught. The solution of the problems in the book should be supplemented by actual practice in making out bills on paper, receipting them, etc. The items included in the several bills in the arithmetic may be used as items in bills against other parties, and several new bills may be made from the first, or third, or fourth, or fifth bill, by using two or three items in one bill, two or three in another, etc.

The pupils should also make out bills relating to common business transactions in the community, including those with a single item, as a bill for several days' work; for digging a well or making a cistern; for repairing a wagon, plow, or other utensil; for a number of pounds of butter; for a number of bushels of wheat, corn, oats, apples, etc.; for a number of sheep, hogs, cattle, etc.

DENOMINATE NUMBERS.

Introduce each lesson in the arithmetic by such oral instruction as will make the pupils familiar with the value and use of the several denominations. This introductory instruction is fully sketched on pages 43–51 inclusive. If previously given, it should be reviewed.

The weakness of the old rule method is very manifest in teaching the reductions of denominate numbers. The pupil who has learned to make these reductions by following the rules for "Reduction Ascending" and "Reduction Descending," is seldom certain that his solution of a problem is correct. When the problem is placed *under* the rule by which it is to be solved, he may proceed with confidence; but give him an independent problem, and he finds difficulty in determining whether the reduction to be made is *ascending* or *descending*. The rules do not assist him in taking the first step. Give the rule pupil a large denominate number to reduce to a lower denomination, as 480 bushels to quarts, and he will almost invariably *divide;* give him a small number to reduce to a higher denomination, and he will as invariably *multiply*.

The analytic method trains the pupil to determine the reduction to be made by a common-sense analysis, so that the first, and each succeeding step, is taken with confidence and certainty. The oral exercises in each of the lessons in denominate numbers should be most faithfully taught, and the written problems should be solved by analysis, entirely independent of the rule. Neither teacher nor pupil should use the rule in solving the problems. When the pupil can make any required reduction promptly and accurately, he may then give attention to the rules.

To secure accuracy, require the pupil to write after each result the abbreviation or sign of its denomination. See Intermediate Arithmetic, pages 146 and 147.

The objection sometimes made to this requirement is a mere cavil. It is true that the multiplication of bushels by 4 does not produce pecks, but it is not true that bushels are multiplied by 4. Take the solution of problem 22, page 146, for illustration. The "48 pk" may be the product of 4 pk. × 12, or 12 pk. × 4. *The actual process is not indicated by the written form.* The mental analysis may be, "Since there are 4 pecks in 1 bushel, in 12 bushels there are 12 times 4 pecks, or 48 pecks." Or, it may be this, "Since there are 4 pecks in 1 bushel, in 12 bushels there are 4 times as many pecks as there are bushels, or 4 times 12 pecks, which is 48 pecks."

The objection may be wholly obviated by placing a comma (,) between the result and its abbreviation or sign, as below:

$$\begin{array}{rl} 5 & \text{bu. 2 pk. 7 qt.} \\ \underline{4} & \\ 20, & \text{pk.} \\ \underline{2} & \\ 22, & \text{pk.} \\ \underline{8} & \\ 176, & \text{qt.} \\ \underline{7} & \\ 183, & \text{qt.} \end{array}$$

$$\begin{array}{l} 8)\overline{183,}\ \text{qt.} \\ 4)\overline{\ 22,}\ \text{pk.} + 7\ \text{qt.} \\ \quad\ \overline{\ \ \ 5,}\ \text{bu.} + 2\ \text{pk.} \\ 5\ \text{bu. 2 pk. 7 qt.}, \textit{Ans.} \end{array}$$

PERCENTAGE.

The revised edition of the Intermediate Arithmetic (1876) contains an elementary treatment of the subject of percentage, with its more common and useful applications. It is designed to be a SHORT COURSE for those pupils who do not remain in school long enough to reach the subject in the Complete Arithmetic.

The section presents the three fundamental cases of percentage, each treated in the simplest possible manner, and their practical applications in profit and loss, commission, insurance, taxes, simple interest, discount, and exchange. The subjects of commission, insurance, and taxes, including duties, are treated exclusively by problems. A knowledge of these departments of business can not be imparted to young children by means of definitions. They must receive such instruction from a living teacher who is practically familiar with business forms and usages. For this information the teacher is referred to the COMPLETE ARITHMETIC.

One Method for each Operation.

Elementary instruction in arithmetic should aim *to make the pupil ready and accurate in the use of one method for each operation.* This may not be the shortest method in every case, but, as a rule, the pupil will reach the result sooner by it than by a method that is less familiar. The attempt to use several methods results in hesitation and often in confusion. It is not safe to attempt to take "short cuts" before the beaten path is familiar.

This sound principle in teaching is faithfully embodied in the added section on percentage. *It presents but one process in each case and for each application,* and the process selected is believed to be the best for beginners, and also for practical purposes.

The Six Per Cent Method.

The method employed for computing simple interest is the "Six Per Cent Method." Experience has proved this to be the shortest method *for all classes of interest problems.* A little drill will enable a pupil to find the interest of $1, at 6%, mentally, and usually at a glance. The interest of any sum of money for the time, at 6%, is then found by the simple process of multiplication. When the interest, at 6%, is found, the interest at any other rate is readily computed. See Intermediate Arithmetic, Sec. XII, Less. VII.

This method has the further advantage of being based on the legal rate of interest in many of the States, and also for debts due the United States. The legal rates in the other States are 5%, 7%, 8%, and 10%, which are respectively $\frac{1}{6}$ less, $\frac{1}{6}$ more, $\frac{1}{3}$ more, and $\frac{2}{3}$ more than 6%. The interest at 10% is easily obtained from the interest at 6% by dividing by 6 and removing the separatrix one place to the right.

Notes and Drafts.

It is not enough that the text-book contain the common forms of notes, checks, and drafts. The pupil should be made familiar with these business papers *by actual practice in writing them,* as in the case of bills. See page 74. The nature of these papers can be best taught orally, and in connection with the pupil's practice in writing them. For information, see Complete Arithmetic, pages 196–202.

DRILL PROBLEMS.

The most successful teachers of arithmetic require their pupils to solve problems, in addition to those presented in the book. These new problems are not only dictated in the recitations for class drill, but they are also written upon the blackboard and assigned as a part of the next lesson.

In giving problems for this additional practice, the teacher should be careful to select those that involve no new principle, and are within the comprehension of the pupils. It is a mistake to give intermediate pupils *the difficult problems found in arithmetics designed for more advanced pupils.* The object in assigning new problems is not to worry pupils with difficulties which they can better meet a few months later, but to afford them additional practice in the application of known principles, and to secure increased readiness and skill in computation.

The several series of problems, given in the following pages, are arranged for use in connection with the designated lessons in the Intermediate Arithmetic. They are carefully graded, and will be found numerous enough for class drills. *The pupils should not have access to the answers.* They are given for the convenience of the teacher.

SECTIONS I TO V.

Review.

1. The sum of two numbers is forty thousand four hundred and sixteen, and the less is five thousand and thirty: what is the greater number? *Ans.* 35386.

2. The subtrahend is 40506, and the remainder is 8764: what is the minuend? *Ans.* 49270.

3. From the sum of 504316 and 92493 take their difference. *Ans.* 184986.

4. The product of two numbers is 184752, and the multiplicand is 1283: what is the multiplier? *Ans.* 144.

5. The divisor is 136, the quotient 2526, and the remainder 105: what is the dividend? *Ans.* 343641.

6. The dividend is 11058 and the quotient is 57: what is the divisor? *Ans.* 194.

7. The product is 110040 and the multiplier is 140: what is the multiplicand? *Ans.* 786.

8. Multiply the sum of all the prime numbers between 10 and 30 by 250. *Ans.* 28000.

9. Divide the product of 1728 and 144 by their quotient. *Ans.* 20736.

10. Divide the sum of 2560 and 1280 by their difference. *Ans.* 3.

SECTION VI.—PROPERTIES OF NUMBERS.

1. What are the prime factors of 1728? *Ans.* 2, 2, 2, 2, 2, 2, 3, 3, 3.

2. What is the greatest common divisor of 308 and 630? *Ans.* 14.

3. Find the greatest common divisor of 100, 225, and 375. *Ans.* 25.

4. What is the greatest common divisor of 128, 224, and 320? *Ans.* 32.

5. Find the greatest common divisor of 99 bushels, 261 bushels, and 504 bushels. *Ans.* 9 bu.

6. What is the least common multiple of 16, 24, 96? *Ans.* 96.

7. What is the least common multiple of 125, 250, and 500? *Ans.* 500.

8. What is the least common multiple of 60, 50, 35, 75? *Ans.* 2100.

9. Find the least common multiple of 24, 48, 72, and 216. *Ans.* 432.

10. Find the least common multiple of 144, 18, 45, and 120. *Ans.* 14401.

SECTION VII.—FRACTIONS.

Lesson II.

Reduce to a fraction—

1. $13\frac{5}{6}$:	*Ans.* $\frac{83}{6}$.	6. $325\frac{5}{26}$.	*Ans.* $\frac{8455}{26}$.
2. $16\frac{5}{12}$.	*Ans.* $\frac{197}{12}$.	7. $12\frac{88}{250}$.	*Ans.* $\frac{3088}{250}$.
3. $156\frac{17}{24}$.	*Ans.* $\frac{3761}{24}$.	8. $161\frac{21}{40}$.	*Ans.* $\frac{6461}{40}$.
4. $342\frac{3}{4}$.	*Ans.* $\frac{1371}{4}$.	9. $121\frac{7}{12}$.	*Ans.* $\frac{1459}{12}$.
5. $116\frac{2}{11}$.	*Ans.* $\frac{1278}{11}$.	10. $200\frac{7}{30}$.	*Ans.* $\frac{6007}{30}$.

Lesson III.

Reduce to a whole or mixed number—

1. $\frac{353}{20}$.	*Ans.* $17\frac{13}{20}$.	6. $\frac{1463}{12}$.	*Ans.* $121\frac{11}{12}$.
2. $\frac{2921}{4}$.	*Ans.* $730\frac{1}{4}$.	7. $\frac{6465}{40}$.	*Ans.* $161\frac{25}{40}$.
3. $\frac{1417}{60}$.	*Ans.* $23\frac{37}{60}$.	8. $\frac{1275}{11}$.	*Ans.* $115\frac{10}{11}$.
4. $\frac{8224}{16}$.	*Ans.* 514.	9. $\frac{6015}{30}$.	*Ans.* $200\frac{15}{30}$.
5. $\frac{1613}{90}$.	*Ans.* $17\frac{83}{90}$.	10. $\frac{2185}{250}$.	*Ans.* $8\frac{185}{250}$.

Lesson IV.

Reduce to lowest terms—

1. $\frac{77}{84}$. *Ans.* $\frac{11}{12}$.
2. $\frac{60}{125}$. *Ans.* $\frac{12}{25}$.
3. $\frac{96}{544}$. *Ans.* $\frac{3}{17}$.
4. $\frac{114}{285}$. *Ans.* $\frac{2}{5}$.
5. $\frac{1644}{2192}$. *Ans.* $\frac{3}{4}$.
6. $\frac{182}{196}$. *Ans.* $\frac{13}{14}$.
7. $\frac{720}{940}$. *Ans.* $\frac{36}{47}$.
8. $\frac{960}{1440}$. *Ans.* $\frac{2}{3}$.
9. $\frac{360}{1032}$. *Ans.* $\frac{15}{43}$.
10. $\frac{468}{1184}$. *Ans.* $\frac{117}{296}$.

Lesson V.

Reduce to a common denominator—

1. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$. *Ans.* $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$.
2. $\frac{3}{4}$, $\frac{5}{9}$, and $\frac{7}{12}$. *Ans.* $\frac{27}{36}$, $\frac{20}{36}$, $\frac{21}{36}$.
3. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{7}{8}$. *Ans.* $\frac{60}{120}$, $\frac{80}{120}$, $\frac{96}{120}$, $\frac{105}{120}$.
4. $\frac{3}{8}$, $\frac{3}{4}$, 3, and $2\frac{1}{2}$. *Ans.* $\frac{3}{8}$, $\frac{6}{8}$, $\frac{24}{8}$, $\frac{20}{8}$.
5. $\frac{2}{3}$, $2\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{5}{3}$. *Ans.* $\frac{4}{6}$, $\frac{15}{6}$, $\frac{5}{6}$.

Lesson VI.

1. Add $\frac{1}{2}$, $\frac{3}{8}$, and $\frac{5}{6}$. *Ans.* $1\frac{17}{24}$.
2. Add $\frac{4}{9}$, $\frac{7}{10}$, and $\frac{11}{15}$. *Ans.* $1\frac{79}{90}$.
3. Add $\frac{1}{3}$, $\frac{1}{8}$, and $\frac{3}{10}$. *Ans.* $\frac{91}{120}$.
4. Add $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$. *Ans.* $3\frac{1}{8}$.
5. Add $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{5}{8}$, and $\frac{7}{10}$. *Ans.* $2\frac{11}{40}$.
6. Add $3\frac{1}{4}$, $6\frac{1}{2}$, and $12\frac{3}{8}$. *Ans.* $22\frac{1}{8}$.
7. Add $62\frac{7}{8}$, $102\frac{3}{5}$, and $125\frac{3}{4}$. *Ans.* $291\frac{9}{40}$.
8. Add $\frac{2}{5}$, $\frac{9}{10}$, and $\frac{2}{3}$ of $\frac{3}{8}$. *Ans.* $1\frac{11}{20}$.
9. Add $2\frac{1}{2}$, $\frac{2}{3}$ of $\frac{3}{5}$, $\frac{5}{6}$ of $\frac{6}{7}$. *Ans.* $3\frac{43}{70}$.
10. Add $\frac{3}{5}$ of $\frac{5}{8}$, $\frac{3}{10}$ of $\frac{5}{6}$, and $\frac{1}{6}$ of $\frac{6}{7}$. *Ans.* $\frac{43}{56}$.

Lesson VII.

1. From $\frac{7}{8}$ take $\frac{4}{7}$. *Ans.* $\frac{17}{56}$.
2. From 3 take $\frac{1}{3}$. *Ans.* $2\frac{2}{3}$.

3. From $\frac{5}{9}$ take $\frac{4}{21}$. *Ans.* $\frac{23}{63}$.
4. From $\frac{7}{9}$ take $\frac{3}{5}$. *Ans.* $\frac{8}{45}$.
5. From $\frac{15}{16}$ take $\frac{4}{7}$. *Ans.* $\frac{41}{112}$.
6. From $\frac{2}{3}$ of $\frac{6}{7}$ take $\frac{1}{5}$ of $\frac{10}{11}$. *Ans.* $\frac{30}{77}$.
7. From $\frac{1}{3}$ of $\frac{3}{4}$ take $\frac{3}{10}$ of $\frac{1}{8}$. *Ans.* $\frac{17}{80}$.
8. From $204\frac{1}{6}$ take $93\frac{1}{7}$. *Ans.* $111\frac{1}{42}$.
9. From $47\frac{2}{7}$ take $34\frac{1}{3}$. *Ans.* $12\frac{20}{21}$.
10. From $\frac{2}{3}$ of $19\frac{1}{2}$ take $\frac{2}{5}$ of $7\frac{2}{3}$. *Ans.* $9\frac{14}{15}$.

Lesson VIII.

1. From the sum of $\frac{3}{5}$ and $\frac{5}{8}$ take their difference. *Ans.* $1\frac{8}{40}$.
2. From the sum of $\frac{3}{7}$ and $\frac{5}{6}$ take the sum of $\frac{5}{21}$ and $\frac{2}{3}$. *Ans.* $\frac{5}{14}$.
3. From the sum of $\frac{4}{5}$ and $\frac{2}{3}$ take their difference. *Ans.* $1\frac{1}{3}$.
4. From the sum of $\frac{5}{8}$ and $\frac{2}{3}$ of $2\frac{1}{2}$ take $\frac{2}{5}$ of $3\frac{1}{3}$. *Ans.* $\frac{23}{24}$.
5. From $17\frac{1}{2}$ take the sum of $6\frac{2}{3}$ and $7\frac{1}{4}$. *Ans.* $3\frac{7}{12}$.
6. From the sum of $\frac{3}{4}$, $\frac{8}{9}$, and $\frac{5}{6}$ take $\frac{2}{3}$. *Ans.* $1\frac{29}{36}$.
7. A boy spends $\frac{3}{4}$ of his time in study, $\frac{1}{20}$ of it in labor, and $\frac{1}{10}$ of it in sleep: what part has he left? *Ans.* $\frac{1}{10}$.
8. A grocer bought $\frac{7}{8}$ of a hogshead of sugar and sold $\frac{1}{4}$ of a hogshead to A, $\frac{1}{3}$ to B, and $\frac{1}{6}$ to C: what part of a hogshead of sugar had he left? *Ans.* $\frac{1}{8}$.

Lesson IX.

Multiply—

1. $\frac{8}{16}$ by 7. *Ans.* $3\frac{1}{2}$.
2. $\frac{15}{16}$ by 25. *Ans.* $23\frac{7}{16}$.
3. $1\frac{7}{11}$ by 5. *Ans.* $8\frac{2}{11}$.
4. $3\frac{2}{3}$ by 8. *Ans.* $29\frac{1}{3}$.
5. $\frac{2}{5}$ of $2\frac{2}{3}$ by 10. *Ans.* $10\frac{2}{3}$.
6. $12\frac{1}{2}$ by 8. *Ans.* 100.

Lesson X.

Multiply—

1. 15 by $\frac{3}{4}$. *Ans.* $11\frac{1}{4}$.
2. 57 by $\frac{5}{8}$. *Ans.* $35\frac{5}{8}$.
3. 435 by $16\frac{2}{3}$. *Ans.* 7250.
4. 329 by $5\frac{1}{2}$. *Ans.* $1809\frac{1}{2}$.
5. 5280 by $\frac{1}{5}$ of $\frac{3}{10}$. *Ans.* $316\frac{4}{5}$.
6. 104 by $18\frac{1}{3}$. *Ans.* $1906\frac{2}{3}$.

Lesson XI.

Multiply—

1. $\frac{3}{4}$ by $\frac{7}{9}$. *Ans.* $\frac{7}{12}$.
2. $\frac{7}{12}$ by $\frac{4}{21}$. *Ans.* $\frac{1}{9}$.
3. $2\frac{1}{2}$ by $1\frac{1}{3}$. *Ans.* $3\frac{1}{3}$.
4. $\frac{5}{8}$ by $3\frac{1}{3}$. *Ans.* $2\frac{1}{12}$.
5. $\frac{8}{11}$ by $\frac{1}{2}$ of $\frac{3}{4}$. *Ans.* $\frac{3}{11}$.
6. $5\frac{1}{4}$ by $\frac{1}{6}$. *Ans.* $\frac{7}{8}$.
7. $\frac{4}{7}$ of 12 by $\frac{7}{8}$. *Ans.* 6.
8. $7\frac{1}{2}$ by $9\frac{1}{4}$. *Ans.* $69\frac{3}{8}$.
9. $\frac{1}{4}$ of $\frac{5}{8}$ by $3\frac{1}{3}$. *Ans.* $\frac{25}{48}$.
10. $\frac{2}{3}$ of $\frac{3}{5}$ by $\frac{2}{5}$ of $\frac{10}{11}$. *Ans.* $\frac{8}{55}$.

Lesson XII.

Divide—

1. $\frac{18}{35}$ by 9. *Ans.* $\frac{2}{35}$.
2. $\frac{15}{32}$ by 20. *Ans.* $\frac{3}{128}$.
3. $3\frac{1}{3}$ by 5. *Ans.* $\frac{2}{3}$.
4. $6\frac{2}{5}$ by 8. *Ans.* $\frac{4}{5}$.
5. $10\frac{3}{4}$ by 12. *Ans.* $\frac{43}{48}$.
6. $16\frac{2}{3}$ by 25. *Ans.* $\frac{2}{3}$.
7. $7\frac{1}{2}$ by 6. *Ans.* $1\frac{1}{4}$.
8. $26\frac{1}{2}$ by 3. *Ans.* $8\frac{5}{6}$.
9. $33\frac{1}{3}$ by 8. *Ans.* $4\frac{1}{6}$.
10. $38\frac{3}{5}$ by 75. *Ans.* $\frac{193}{375}$.

Lesson XIII.

Divide—

1. 6 by $\frac{4}{5}$. *Ans.* $7\frac{1}{2}$.
2. 14 by $\frac{7}{8}$. *Ans.* 16.
3. 24 by $\frac{5}{8}$. *Ans.* $38\frac{2}{5}$.
4. 32 by $\frac{5}{6}$. *Ans.* $38\frac{2}{5}$.
5. 41 by $\frac{2}{3}$. *Ans.* $61\frac{1}{2}$.
6. 45 by $2\frac{1}{3}$. *Ans.* $19\frac{2}{7}$.
7. 50 by $6\frac{1}{4}$. *Ans.* 8.
8. 100 by $16\frac{2}{3}$. *Ans.* 6.
9. 120 by $4\frac{1}{6}$. *Ans.* $28\frac{4}{5}$.
10. 225 by $16\frac{2}{3}$. *Ans.* $13\frac{1}{2}$.

Lesson XIV.

Divide—

1. $\frac{1}{3}$ by $\frac{1}{2}$. *Ans.* $\frac{2}{3}$.
2. $\frac{4}{7}$ by $\frac{2}{3}$. *Ans.* $\frac{6}{7}$.
3. $\frac{5}{12}$ by $\frac{3}{4}$. *Ans.* $\frac{5}{9}$.
4. $\frac{11}{15}$ by $\frac{2}{3}$. *Ans.* $1\frac{1}{10}$.
5. $\frac{21}{25}$ by $\frac{3}{10}$. *Ans.* $2\frac{4}{5}$.
6. $3\frac{1}{6}$ by $\frac{1}{2}$. *Ans.* $6\frac{1}{3}$.
7. $4\frac{1}{5}$ by $\frac{2}{3}$. *Ans.* $6\frac{3}{10}$.
8. $\frac{2}{3}$ by $4\frac{1}{5}$. *Ans.* $\frac{10}{63}$.
9. $6\frac{1}{3}$ by $3\frac{1}{3}$. *Ans.* $1\frac{9}{10}$,
10. $\frac{3}{5}$ by $\frac{2}{3}$ of $\frac{6}{7}$. *Ans.* $1\frac{1}{20}$.

Lesson XVI.

1. 25 is $\frac{5}{7}$ of what number? *Ans.* 35.
2. 56 is $\frac{2}{3}$ of what number? *Ans.* 84.
3. Reduce $\frac{242}{726}$ to its lowest terms. *Ans.* $\frac{1}{3}$.
4. Reduce $245\frac{5}{12}$ to a fraction. *Ans.* $\frac{2945}{12}$.
5. Reduce $\frac{5780}{325}$ to a mixed number. *Ans.* $17\frac{51}{65}$.
6. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{5}{6}$ to the same denominator. *Ans.* $\frac{15}{30}$, $\frac{10}{30}$, $\frac{12}{30}$, $\frac{25}{30}$.
7. Add $\frac{2}{5}$, $\frac{3}{4}$, and $\frac{5}{6}$. *Ans.* $1\frac{59}{60}$.
8. From $\frac{7}{8}$ take $\frac{5}{12}$. *Ans.* $\frac{11}{24}$.
9. Add $\frac{2}{3}$, $\frac{2}{3}$ of $\frac{3}{5}$, and $2\frac{1}{2}$. *Ans.* $3\frac{17}{30}$.
10. Add $\frac{2}{3}$ of $\frac{5}{6}$, $\frac{1}{2}$ of $\frac{7}{8}$, and $\frac{2}{5}$ of $2\frac{1}{2}$. *Ans.* $1\frac{143}{144}$.
11. Add $26\frac{3}{4}$, $15\frac{5}{8}$, and $12\frac{5}{6}$. *Ans.* $55\frac{5}{24}$.
12. From $\frac{3}{4}$ of $2\frac{1}{3}$ take $\frac{1}{2}$ of $\frac{2}{3}$. *Ans.* $1\frac{5}{12}$.
13. From $17\frac{3}{4}$ take $\frac{2}{5}$ of $6\frac{1}{4}$. *Ans.* $15\frac{1}{4}$.
14. From the sum of $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{5}{12}$ take the sum of $\frac{2}{3}$ and $\frac{3}{4}$. *Ans.* $\frac{17}{24}$.
15. Multiply $\frac{3}{4}$ by $2\frac{1}{3}$. *Ans.* $1\frac{3}{4}$.
16. Multiply $\frac{3}{4}$ of $\frac{2}{3}$ by $\frac{7}{8}$ of $\frac{5}{7}$. *Ans.* $\frac{5}{16}$.
17. Multiply $\frac{1}{3}$ of $3\frac{1}{2}$ by $\frac{1}{4}$ of $5\frac{1}{2}$. *Ans.* $1\frac{29}{48}$.
18. Multiply $6\frac{2}{3}$ by $12\frac{1}{2}$. *Ans.* $83\frac{1}{3}$.
19. Divide $\frac{7}{8}$ by $\frac{3}{5}$. *Ans.* $1\frac{11}{24}$.
20. Divide $17\frac{1}{2}$ by $\frac{3}{5}$. *Ans.* $29\frac{1}{6}$.
21. Divide $7\frac{5}{6}$ by $3\frac{2}{3}$. *Ans.* $2\frac{3}{22}$.

22. Divide $\frac{1}{2}$ of $\frac{5}{6}$ of $\frac{3}{4}$ by $2\frac{1}{3}$. *Ans.* $\frac{15}{112}$.

23. Divide $\frac{3}{5}$ of $6\frac{1}{4}$ by $\frac{2}{3}$ of $7\frac{1}{2}$. *Ans.* $\frac{3}{4}$.

24. Divide $\frac{3}{4}$ of $\frac{4}{5}$ by $\frac{2}{3}$ of $2\frac{1}{10}$. *Ans.* $\frac{3}{7}$.

25. Divide the sum of $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ by the product of $\frac{3}{4}$ and $\frac{2}{3}$. *Ans.* $4\frac{11}{12}$.

26. At $\frac{3}{8}$ of a dollar a yard, how many yards can be bought for $7\frac{1}{2}$? *Ans.* 20 yd.

27. What will $18\frac{3}{4}$ pounds of butter cost at $17\frac{1}{2}$ cents per pound? *Ans.* $3.28\frac{1}{8}$.

28. What will $6\frac{1}{2}$ pounds of beef cost at $6\frac{1}{2}$ cents per pound? *Ans.* $.42\frac{1}{4}$.

29. How many pounds of butter, at $18\frac{3}{4}$ cents a pound, can be bought with $5? *Ans.* $26\frac{2}{3}$ lb.

30. What is the cost of four fields, containing respectively $4\frac{1}{3}$, $2\frac{1}{2}$, $3\frac{2}{3}$, and $1\frac{8}{16}$ acres, at $25 an acre? *Ans.* $300.

SECTION VII.—DECIMAL FRACTIONS.

Lesson I.

1. Write in words .040603.

2. Write in words 40.04045.

3. Write in words 4000.004.

4. Write in words 4000000.000004.

Express in figures:

5. Seven hundred and forty-five ten-thousandths.

6. Fifteen thousand and eighty-five millionths.

7. Four hundred and five units and five thousand and sixty-two hundred thousandths.

8. Three hundred and sixteen ten-thousandths.

9. Nine thousand and seven units and nine hundred and seven thousandths.

10. Three thousand units and three thousand and eighteen millionths.

Lesson II.

Reduce to a common fraction—

1. .375.	*Ans.* $\frac{3}{8}$.	4. .085.	*Ans.* $\frac{17}{200}$.
2. .72.	*Ans.* $\frac{18}{25}$.	5. .0025.	*Ans.* $\frac{1}{400}$.
3. .0075.	*Ans.* $\frac{3}{400}$.	6. .03125.	*Ans.* $\frac{1}{32}$.

Change to a decimal—

7. $\frac{8}{25}$.	*Ans.* .32.	12. $\frac{7}{120}$.	*Ans.* $.058\frac{1}{3}$.
8. $\frac{54}{225}$.	*Ans.* .24.	13. $15\frac{3}{20}$.	*Ans.* 15.15.
9. $\frac{21}{40}$.	*Ans.* .525.	14. $40\frac{13}{125}$.	*Ans.* $40.10\frac{2}{5}$.
10. $\frac{4}{75}$.	*Ans.* $.05\frac{1}{3}$.	15. $62\frac{5}{80}$.	*Ans.* 62.0625.
11. $\frac{29}{500}$.	*Ans.* .058.	16. $54\frac{5}{12}$.	*Ans.* $54.41\frac{2}{3}$.

Lessons III and IV.

1. From the sum of nine thousand and fifty-six millionths and three hundred and eight hundred-thousandths take six hundred and seventy-eight millionths. *Ans.* .011458.

2. From the sum of .70604 and .050435 take their difference. *Ans.* .10087.

3. To the sum of .060018 and .00704 add their difference. *Ans.* .120036.

4. From the sum of 83.0041 and 60.809 take their difference. *Ans.* 121.618.

5. To the sum of 4.0708 and 1.905 add their difference. *Ans.* 8.1416.

Lesson V.

Multiply—

1. .0052 by 5.2. *Ans.* .02704.	3. 77.2 by .0006. *Ans.* .04632.
2. 840 by .025. *Ans.* 21.	4. .0045 by 31.2. *Ans.* .1404.

5. 75 by .033$\frac{1}{3}$. *Ans.* 2.5.

6. 2.16$\frac{2}{3}$ by 1.2. *Ans.* 2.6.

7. .052 by .00015. *Ans.* .0000078.

8. 3.25 by 100. *Ans.* 325.

9. .0084 by 1000. *Ans.* 8.4.

10. .06$\frac{1}{4}$ by 100. *Ans.* 6.25.

Lesson VI.

Divide—

1. 11.05 by .034. *Ans.* 325.

2. .00468 by 3.12. *Ans.* .0015.

5. .0084 by 2.1. *Ans.* .004.

6. 2.796 by 1.16$\frac{1}{2}$. *Ans.* 2.4.

7. 25 by .033$\frac{1}{3}$. *Ans.* 750.

3. 17.5 by .0025. *Ans.* 7000.

4. .0904 by .0565. *Ans.* 1.6.

8. 1.728 by 720. *Ans.* .0024.

9. 15.24 by 100. *Ans.* .1524.

10. 3.25 by 1000. *Ans.* .00325.

Review.

1. From 152 ten-thousandths take 27 millionths. *Ans.* .015173.

2. Change $\frac{5}{80}$ to a decimal and multiply the result by 15 hundredths. *Ans.* .009375.

3. Change 6.025 to a common mixed number. *Ans.* 6$\frac{1}{40}$.

4. Change 62$\frac{5}{40}$ to a mixed decimal. *Ans.* 62.125.

5. Multiply 36$\frac{1}{4}$ by 2.5. *Ans.* 90.625.

6. Multiply 15 hundredths by 15 thousandths and from the product take 15 millionths. *Ans.* .002235.

7. Divide 256 thousandths by 16 millionths. *Ans.* 16000.

8. Divide 22 hundredths by 55 ten-thousandths. *Ans.* 40.

9. Divide 36 thousandths by 60 units, and multiply the quotient by 100. *Ans.* .06.

10. Change $\frac{3}{250}$ to a decimal and divide the result by 2½. *Ans.* .0048.

SECTION IX.—UNITED STATES MONEY.

Miscellaneous Problems.

1. Reduce $75 to mills. *Ans.* 75000*m.*

2. Reduce $5 to cents. *Ans.* 500*c.*

3. Reduce 45000 mills to dollars. *Ans.* $45.

4. Reduce 10400 cents to dollars. *Ans.* $104.

5. Reduce $10.62½ cents to mills. *Ans.* 10625*m.*

6. From the sum of $90.62½ and $8.12½ take $15.35. *Ans.* $83.40.

7. To the sum of $6.37½ and $3.06¼ add their difference. *Ans.* $12.75.

8. A lady bought a dress for $8.50, a pair of gloves for $2.25, and a hat for $6.40, and gave the clerk $20: how much money did she receive as change? *Ans.* $2.85.

9. What will 12½ dozens of eggs cost at 18 cents a dozen? *Ans.* $2.25.

10. A carpenter earns $3.37½ a day: how much will he earn in 22 days? *Ans.* $74.25.

11. A man paid $165 for 40 sheep: how much did they cost a head? *Ans.* $4.12½.

12. At 16⅔ cents a dozen, how many dozens of eggs can be brought for $5? *Ans.* 30 doz.

13. At $7.50 a ton, how many tons of coal can be bought for $101.25? *Ans.* 13½ tons.

14. At 37½ cents a bushel, how many bushels of oats can be bought for $19.25? *Ans.* 51⅓ bu.

15. Multiply $3.33⅓ by 15 and divide the product by $.62½. *Ans.* 80.

16. A man worked 52 weeks, at $12.25 a week, and paid $2.62½ a week for board: what were his net earnings? *Ans.* $500.50.

17. A man bought 12 cows, at $35 a head, hired them pastured 6 weeks at $.37½ each per week, and then sold them at $40 a head: how much did he gain? *Ans.* $33.

SECTION X.—DENOMINATE NUMBERS.

Lessons I and II.

1. How many pints in 3 pecks? *Ans.* 48 pt.
2. How many quarts in ¾ of a bushel? *Ans.* 24 qt.
3. How many pints in 2½ bushels? *Ans.* 160 pt.
4. How many pecks in 20 bushels? *Ans.* 80 pk.
5. How many bushels in 84 quarts? *Ans.* 2 bu. 2 pk. 4 qt.
6. Reduce 5 bu. 6 qt. to pints. *Ans.* 332 pt.
7. Reduce 73 quarts to bushels. *Ans.* 2 bu. 1 qt. 1 pt.
8. Reduce 84 pecks to quarts. *Ans.* 672 qt.
9. How many quarts in 3½ gallons? *Ans.* 14 qt.
10. How many pints in 15 gallons? *Ans.* 120 pt.
11. How many gallons in 60 pints? *Ans.* 7 gal. 2 qt.
12. How much vinegar in a half-barrel? *Ans.* 15 gal. 3 qt.

13. At 20 cents a quart, how many quarts of oil can be bought for $10? How many gallons? *Ans.* 12 gal. 2 qt.

14. How many bottles, each holding 3 pints, can be filled from a barrel of syrup? *Ans.* 84.

15. A boy sold 2½ bushels of strawberries, at 12½ cents a quart: how much did he receive? *Ans.* $10.

16. A man bought 40 barrels of apples, 2¾ bushels each, at $1.75 a barrel, and sold the apples at $.75 a bushel: how much did he gain? *Ans.* $12.50.

Lessons III—VI.

1. Reduce 120 rods to feet. *Ans.* 1980 ft.

2. Reduce 990 feet to rods. *Ans.* 60 rods.

3. Reduce 1 mi. 48 rd. 2 ft. to feet. *Ans.* 6074 ft.

4. Reduce 17160 feet to miles. *Ans.* 3 mi. 2 fur.

5. Reduce 243 square feet to square yards. *Ans.* 27 sq. yd.

6. Reduce 520 perches to acres. *Ans.* 3 A. 1 R.

7. Reduce 1000 cubic yards to cubic feet. *Ans.* 27000 cu. ft.

8. Reduce 1000 cubic feet to cubic yards. *Ans.* 37 cu. yd. 1 cu. ft.

9. How many steps, of 2 ft. 3 in. each, will a boy take in walking ½ mile? *Ans.* 1173⅓.

10. How many cubic inches in a box 1 ft. 6 in. long, 1 ft. 2 in. wide, and 10 in. deep. *Ans.* 2520 cu. in.

Lessons VII and VIII.

1. Reduce 12° 24′ to minutes. *Ans.* 744′.

2. Reduce 320′ to degrees. *Ans.* 5° 20′.

3. Reduce 120 days to hours. *Ans.* 2880 h.

4. Reduce 1 w. 3 da. 6 h. to minutes. *Ans.* 14760 min.

5. How many minutes in ⅔ of a day? *Ans.* 960 min.

6. How many hours in the first three months of 1876? *Ans.* 2184 h.

7. If a man's pulse beat 70 times in a minute, how many times will it beat in 1 day? *Ans.* 100800.

8. If a clock ticks 4 times a second, how many times will it tick in ½ of a day? *Ans.* 172800.

9. If a girl read 30 minutes each morning, how many hours will she read in three summer months? *Ans.* 46 hours.

10. If a man work 8 hours each day, Sundays, New Years, Fourth of July, and Christmas excepted, how many hours will he work in the year 1876? *Ans.* 2488 h.

Lessons IX—XI.

1. Reduce 256 ounces to drams. *Ans.* 4096 dr.

2. Reduce 256 ounces to pounds. *Ans.* 16 lb.

3. Reduce 5 lb. 10 oz. 3 dr. to drams. *Ans.* 1443 dr.

4. Reduce 500 drams to pounds. *Ans.* 1 lb. 15 oz. 4 dr.

5. What will 2½ barrels of flour cost, at 5 cents a pound? *Ans.* $24.50.

6. What will 375 pounds of hay cost, at 80 cents a hundred? *Ans.* $3.00.

7. A man bought 8 tons of hay at $12.50 a ton, and sold it at $.75 a hundred pounds: how much did he gain? *Ans.* $20.

8. How many pounds in 14 ingots of silver, each weighing 550 ounces? *Ans.* 641⅔ lbs.

9. A lady bought a watch-chain weighing 1 oz. 8 pwt., at $1.75 a pwt.: what was the cost of the chain? *Ans.* $49.

10. How many dozens of 2-grain quinine pills can be made from 1 ℥ 2 ʒ of quinine? *Ans.* 25 doz.

Lesson XIV.

1. How many half-pint bottles will 2½ gallons of olive-oil fill? *Ans.* 40.

2. A boy bought 3 pecks of chestnuts, at 50 cents a peck, and sold them at 5 cents a pint: how much did he gain? *Ans.* 90 cts.

3. How many quart-baskets will 2¼ bushels of strawberries fill? *Ans.* 72.

4. A dealer bought 5 gross of steel pens at $1.50 a gross, and sold them at 15 cents a dozen: how much did he gain? *Ans.* $1.50.

5. A dealer bought 10 reams of note paper at $2.80 a ream, and sold it at 20 cents a quire: how much did he gain? *Ans.* $12.

6. How many times will a clock that ticks twice a second, tick in the month of May? *Ans.* 5356800.

7. How many silver spoons, each weighing 1 oz. 10 pwt., can be made of 2 lb. 6 oz. of silver? *Ans.* 20.

8. If a horse eat 1 pk. 4 qt. of oats in a day, how many bushels will he eat in the months of September, October, and November? *Ans.* 34 bu. 4 qt.

9. How many bricks, 8 inches long and 4 inches wide, will pave a walk 30 feet long and 2 ft. 4 in. wide? *Ans.* 315.

10. How many times will a wheel 6 ft. 8 in. in circumference rotate in running ⅔ of a mile? *Ans.* 528.

11. How many lengths of fence, 11 ft. each, will enclose a rectangular field 40 rd. long and 30 rd. wide? *Ans.* 210.

12. How many square yards in the floor of a room 18 feet long and 12½ feet wide? *Ans.* 25 sq. yd.

13. How many square yards in a piece of carpet 60 yards long and ¾ of a yard wide? *Ans.* 45 sq. yd.

14. How many yards of carpet, ¾ of a yard wide, will cover a floor 21 feet long and 15 feet wide? *Ans.* 46⅔ yd.

15. What will it cost to lay a pavement 36 ft. long and 9 ft. 6 in. wide, at 40 cents per square yard? *Ans.* $15.20.

16. How many square feet of lumber in 10 boards, each 12 feet long and 8 inches wide? *Ans.* 80 sq. ft.

17. How many acres in a tract of land 2 miles square? *Ans.* 2560 A.

18. What is the difference between 3 feet square and 3 square feet? *Ans.* 6 sq. ft.

19. How many cords of four-foot wood in a pile 36 feet long and 7⅑ feet high? *Ans.* 8 cords.

20. What will it cost to dig a cellar 27 feet long, 15 feet wide, and 5⅓ feet deep, at $1.25 a cubic yard? *Ans.* $100.

SECTION XI.—COMPOUND NUMBERS.

Lessons I—IV.

1. To the sum of 13 bu. 3 pk. 5 qt. and 7 bu. 1 pk. 1 pt. add their difference. *Ans.* 27 bu. 3 pk. 2 qt.

2. From the sum of 3 mi. 5 fur. 29 rd. 2 yd. and 1 mi. 32 rd. 1 yd. take their difference.
Ans. 2 mi. 1 fur. 24 rd. 2 yd.

3. Multiply the sum of 30 lb. 13 oz. 15 dr., 9 lb. 4 oz. 12 dr., and 2 lb. 14 dr., by 8. *Ans.* 337 lb. 12 oz. 8 dr.

4. Multiply the difference between 45 A. 1 R. 16 P. and 9 A. 3 R. and 19 P., by 15. *Ans.* 532 A. 0 R. 35 P.

5. From 5 mi. 2 fur. 20 rd. 4 yd. 1 ft. 5 in. take 2 mi. 34 rd. 2 yd. 2 ft. 7 in.
Ans. 3 mi. 1 fur. 26 rd. 1 yd. 1 ft. 10 in.

6. Add 30 gal. 3 qt. 1 pt., 7 gal. 2 qt., and 15 gal. 1 pt., and divide the sum by 6. *Ans.* 8 gal. 3 qt. $1\frac{1}{3}$ pt.

7. Boston is 71° 4′ 9″ W. longitude and San Francisco is 122° 26′ 15″ W. longitude: how much farther west is San Francisco than Boston? *Ans.* 51° 22′ 6″.

8. A note was dated July 20, 1871, and was paid May 5, 1876: how long did it run? *Ans.* 4 yr. 9 mo. 15 da.

9. What is the distance round a rectangular field which is 24 rd. 3 yd. 1 ft. in length, and 14 rd. 2 yd. 1 ft. 6 in. in width. *Ans.* 78 r. 2 ft.

SECTION XII.—PERCENTAGE.

Lessons I—IV.

1. Express decimally 3%; $\frac{2}{5}$%; $\frac{2}{7}$%; $\frac{1}{20}$%.
Ans. .03; .004; .00$\frac{2}{7}$; .0005.

2. What is $\frac{3}{8}$% of 52.8 lb? *Ans.* .198 lb.

3. What is $2\frac{1}{4}$% of $\frac{3}{4}$ of $3252.60? *Ans.* $54.887+.

4. What per cent of 62.5 is 7.5? *Ans.* 12%.

5. What per cent of 37½ is 6¼? *Ans.* 16⅔%.

6. $187.50 is 12% of what sum of money? *Ans.* $15.625.

7. $565.785 is 110% of what sum of money? *Ans.* $514.35.

8. $2195.475 is 100¼% of what sum of money? *Ans.* $2190.

9. $101.20 is 62½% of what sum? *Ans.* $161.92.

10. A school enrolls 45 pupils, and the average number in daily attendance is 41: what is the per cent of attendance? *Ans.* $91\frac{1}{9}$%.

11. The average number of pupils in daily attendance at a school is 219, which is 91¼% of the number enrolled: what is the enrollment? *Ans.* 240.

12. A pupil answered correctly 65 of the 80 questions submitted at an examination: what per cent of the questions did he answer correctly? *Ans.* 81¼%.

Lessons V and VI.

1. A horse which cost $140, was sold for $156: what was the per cent of gain? *Ans.* $11\frac{3}{7}$%.

2. A merchant sold silk at $1.74 a yard and gained 16%: what was the cost? *Ans.* $1.50.

3. A watch was sold for $210, which was 25% less than its cost: what was the cost? *Ans.* $280.

4. A grocer bought sugar at $8.75 per hundred pounds, and sold it at 10 cents per pound: what was the gain per cent? *Ans.* $14\frac{2}{7}$%.

5. A clerk sold 2¾ yards of broadcloth, which was marked at $2.80 a yard, for 20% off: how much did he receive for the cloth? *Ans.* $6.16.

6. A man lost $5 and had $7.50 remaining: what per cent of his money did he lose? *Ans.* 40%.

7. A house, valued at $3240, is insured for ⅔ of its value, at ⅝%: what is the annual premium? *Ans.* $13.50.

8. A paper mill is insured for $9840, at 1¼%: what is the annual premium? *Ans.* $123.

9. An agent sold $1650 worth of school furniture and received 12½% commission: how much commission did he receive? *Ans.* $206.25.

10. A commission merchant sold 5000 bushels of wheat, at $1.62½ a bushel, and charged 5% commission: what was his commission? How much did he remit as net proceeds of the sale? *Ans.* { $406.25, commission.
$7718.75, proceeds.

Lessons VI—VIII.

1. What is the interest of $1 for 3 mo. 3 da. at 6%? *Ans.* $.0155.

2. What is the interest of $1 from May 1, 1874, to January 10, 1876, at 6%? *Ans.* $.1015.

3. What is the interest of $42.50 for 1 yr. 15 da., at 8%? *Ans.* $3.541+.

4. What is the interest of $85.60 from June 12, 1874, to August 6, 1876, at 5%? *Ans.* $9.202.

5. A note of $250, dated June 12, 1874, was paid May 3, 1875, with interest at 9%: what was the amount? *Ans.* $270.06.

6. A man borrowed $5640 for 2 yr., 5 mo., at 6%, and then loaned the money for the same time, at 7½%: what did he gain? *Ans.* $204.45.

7. A note, dated January 20, 1876, was paid May 3, 1876: how many days did it run? *Ans.* 104 days.

8. A note of $75, dated April 10, 1876, was paid July 3, 1876, with interest at 8% for the time in days, without grace: what was the amount? *Ans.* $76.40.

9. A note of $120, payable in 45 days, was discounted by a bank, at 8%: what were the proceeds? *Ans.* $118.72.

10. A note of $85.60, payable July 3, 1876, was discounted by a bank April 10, 1876, at 10%: what were the proceeds? *Ans.* $83.53+.

11. $450. COLUMBUS, O., *June* 5, 1875.

Sixty days after date, I promise to pay to John Davis, or order, four hundred and fifty dollars, for value received.

CHARLES GREEN.

What will be the proceeds of the above note, discounted by a bank at 9% per annum? *Ans.* $442.9125.

SECTION XIII.—MENSURATION.

1. How many square yards in the walls of a room 21 feet long, 15 feet wide, and 10 feet high? *Ans.* 80 sq. yd.

2. How many square inches in a triangle whose base is 13½ inches and altitude 10 inches? *Ans.* 67½ sq. in.

3. How many square inches in a circular plate of glass 10 inches in diameter? *Ans.* 78.54 sq. in.

4. What is the circumference of a globe 15 inches in diameter? *Ans.* 47.124 ft.

5. How many square feet of tin will line a bin 4 feet long, 2½ feet wide, and 3 feet deep? *Ans.* 59 sq. ft.

6. How many square feet of lumber in 50 boards 12 ft. long and 10 in. wide, and 30 planks 16 ft. long, 9 in. wide, and 1¾ inches thick? *Ans.* 1130 sq. ft.

7. How many bricks, each occupying 80 cubic inches, will be required to build a wall 120 ft. long, 1 ft. thick, and 8 ft. high? *Ans.* 20736.

8. How many cubic feet in a round log of wood 12 ft. long, and 2½ ft. in diameter (average)? *Ans.* 58.905 cu. ft.

9. How many gallons of water will fill a cask 3 feet long and 1 ft. 8 in. in diameter? *Ans.* 48.96 gal.

10. How many square inches of gold-leaf will cover a ball 5 inches in diameter? *Ans* 78.54 sq. in.

THE COMPLETE ARITHMETIC.

THE COMPLETE ARITHMETIC is designed for pupils who are familiar with the elements of numbers, presented in the lower books of the series. IT SHOULD NOT BE PUT INTO THE HANDS OF BEGINNERS.

It is called "Complete" because it includes *all* the subjects which properly belong to a school arithmetic, and also because it fills out, or *completes*, the course of instruction. It is not complete in the sense of presenting a course of instruction adapted to all grades of pupils. It treats all subjects from the stand-point of pupils who have acquired a knowledge of the elementary operations, with both simple and denominate numbers.

The sections devoted to the fundamental rules present only a review of the processes, principles, and definitions, with the more useful contractions. The pupil who has not acquired some skill in these elementary combinations of numbers will need much more practice than is here presented. The section on Denominate Numbers likewise requires a good degree of familiarity with the tables and with the processes of reduction.

The Complete Arithmetic is, in brief, designed to follow and supplement the Intermediate, and the best results will be attained by using the two books in order. This will also be true economy. The great majority of the pupils who begin the study of arithmetic, do not attend school long enough to do more than master the Intermediate Arithmetic—the smaller and cheaper book.

An Abridged Course.

The Complete Arithmetic embraces a few subjects which may well be omitted by pupils in country schools, and in graded schools below the high school. Among these are the Equation of Payments, the Equation of Accounts, Compound Proportion, Compound Partnership, and Cube Root with its applications. These subjects may be profitably studied by advanced pupils who intend to be accountants or teachers, and hence they are included in the book.

The first 210 pages, with the Problems for Analysis (pp. 239–245), and the General Review and Test Questions (pp. 268–286), constitute a thorough practical course in arithmetic for pupils of the grammar-school grade.

General Method of Instruction.

Since the Complete Arithmetic embodies the same general method of teaching as the Intermediate, it is not deemed necessary to repeat the suggestions and explanations already submitted. The teacher is referred to pages 53 to 78, inclusive, for instructions respecting the proper use of the mental problems, the induction of the written processes from the oral, the induction and statement of principles, definitions, and rules, the dictation of problems for additional drill, etc.

Abbreviated Processes.

Much attention is given in the Complete Arithmetic to those abbreviated processes which the skillful arithmetician can frequently use with advantage. These contractions are best learned after the pupil is familiar with the full or regular processes, and hence they are presented in this higher book.

Special attention is called to four contractions in multiplication (pp. 22–24) and division (pp. 28–32), and to cancellation (pp. 35, 36). The second contraction in multiplication and the third in division are constantly used by accountants and business men, and yet comparatively few pupils ever employ them in school work. This is well in the case of primary and intermediate pupils, but those in the more advanced classes should be trained to see and use "short cuts." It is a waste of time for an arithmetician to multiply or divide a number by $12\frac{1}{2}$, $16\frac{2}{3}$, $33\frac{1}{3}$, 25, 125, 250, etc., by the usual full method. These contractions are much employed in percentage, and especially in the mental problems.

The process of cancellation is employed, with a great gain in time, in the reduction of compound and complex fractions, in the multiplication and division of fractions, in proportion and mensuration, and in all other processes which involve the division of the product of several numbers by the product of several other numbers. Advanced pupils should be taught to indicate operations and cancel whenever this can be done with advantage.

It is not wise to spend time in teaching contractions which pupils will have little opportunity to use. It requires continued practice to enable one to use an abbreviated process skillfully and with consequent advantage, and the practical value of such a process depends on one's ability to use it.

Different Processes.

The Intermediate Arithmetic gives but one process and generally but one explanation. The Complete Arithmetic, being designed for more advanced pupils, gives two processes when more than one is in use. This enables the pupil to use the process which may be preferred, and, at the same time, he learns that this is not the only approved one.

It is not intended that pupils shall be made equally familiar with these different processes. *The one preferred should be thoroughly mastered for use;* the other may be passed over when understood.

In teaching advanced pupils it is often best to give more than one solution of a problem and more than one explanation of a process. The solution or explanation adapted to one class of pupils may not be equally well adapted to another. When, however, one solution or explanation is clearly understood by all the pupils in a class, it is not necessary to give another, and special care should always be taken not to confuse pupils by giving successively different solutions and explanations. The desire of the teacher to tell all he knows, is sometimes exercised at the expense of the pupil.

Illustrative Solutions.

Part II contains illustrative solutions of all the more difficult written problems. These solutions are designed to present good models of slate and blackboard work, and much pains has been taken to give the best possible forms. The explanations added to the solutions are designed for the teacher, but, in most cases, they also present what should be given orally by the pupil. The solutions of the mental problems are also presented as models of analysis, and, as such, they will repay a careful study by the teacher.

Test Problems.

The problems in the Complete Arithmetic are so numerous and varied that teachers will not find it necessary to use many additional problems for drill. The lists of easy problems given on pages 78–96 may be dictated for recitation drills on the processes which they embrace, and similar problems for other drills may be easily made up by the teacher or selected from other arithmetics. "Ray's Test Examples" contains a large number of well-selected problems.

PART II.

MODELS OF ANALYSIS, AND ILLUSTRATIVE SOLUTIONS OF THE MORE DIFFICULT PROBLEMS IN WHITE'S GRADED-SCHOOL ARITHMETICS.

INTERMEDIATE ARITHMETIC.

MODELS OF ANALYSIS.

Page 53. (49.)*

7. If a man earn 3 dollars in 1 day, in 6 days he will earn 6 times 3 dollars, which is 18 dollars.

Page 55. (50.)

10. If there are 5 school-days in 1 week, in 6 weeks there are 6 times 5 school-days, which is 30 school-days; in 8 weeks there are 8 times 5 school-days, which is 40 school-days; in 10 weeks there are 10 times 5 school-days, which is 50 school-days.

Page 57. (52.)

4. If one orange cost 8 cents, 6 oranges will cost 6 times 8 cents, which is 48 cents.

Page 58. (53.)

6. If 5 men can build a wall in 7 days, it will take 1 man 5 times 7 days, or 35 days, to build it.

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* Figures in curves refer to edition of 1876.

Page 59. (53.)

7. If a box of crackers last 8 men 7 days, it will last 1 man 8 times 7 days, which is 56 days.

Page 60. (55.)

7. If 4 equal pipes will empty a cistern in 8 hours, 1 pipe will empty it in 4 times 8 hours, which is 32 hours.

Page 62. (57.)

8. If 7 men can do a piece of work in 9 days, it will take 9 times 7 men, or 63 men, to do it in 1 day.

Page 64. (58.)

5. In 1 hour the two men will be 3 miles plus 4 miles, or 7 miles, apart, and if they separate 7 miles in 1 hour, in 8 hours they will separate 8 times 7 miles, or 56 miles.

Page 68. (61.)

7. If one stage goes 9 miles an hour and the other 6 miles, they will be 9 miles less 6 miles, or 3 miles, apart in 1 hour, and in 5 hours they will be 5 times 3 miles, or 15 miles, apart.

Page 68. (62.)

10. If he walked from the city 4 miles an hour for 9 hours, he walked 9 times 4 miles, or 36 miles; if he returned 3 miles an hour for 4 hours, he returned 4 times 3 miles, or 12 miles, and the distance he was then from the city was the difference between 36 miles and 12 miles, which is 24 miles.

Page 70. (64.)

5. If a man can walk 3 miles an hour, it will take him as many hours to walk 15 miles as 3 miles are contained times in 15 miles, which is 5; it will take him 5 hours to walk 15 miles.

Page 71. (65.)

3. If there are 4 trees in a row, 16 trees make as many rows as 4 trees are contained times in 16 trees, which is 4; there are 4 rows of trees in the orchard.

Page 73. (67.)

8. As many plows at $6 each can be bought for $48 as $6 are contained times in $48, which is 8; at $6 each, 8 plows can be bought for $48.

Page 76. (69.)

8. If a man work 8 hours a day, it will take him as many days to work 96 hours, as 8 hours are contained times in 96 hours, which is 12; it will take him 12 days to work 96 hours.

Page 77. (71.)

5. If 1 man can do the work in 90 days, it will take as many men to do it in 9 days, as 9 days are contained times in 90 days, which is 10; it will take 10 men to do the work in 9 days.

6. If a quantity of provisions will last 72 men 1 day, it will last 9 men as many days as 9 men are contained times in 72 men, which is 8; it will last 9 men 8 days.

Page 84. (77.)

5. The value of 24 yards of carpeting, at $2 a yard, is 24 times $2, or $48, and $48 will pay for as many barrels of flour, at $8 a barrel, as $8 are contained times in $48, which is 6; 6 barrels of flour will pay for the carpeting.

9. If 6 men can do a piece of work in 8 days, 1 man can do it in 6 times 8 days, or in 48 days, and if 1 man can do it in 48 days, it will take as many men to do it in 12 days, as 12 days are contained times in 48 days, which is 4; it will take 4 men to do the work in 12 days.

Page 105. (96.)

4. He spent $\frac{1}{2}$ plus $\frac{1}{3}$ of his money; $\frac{1}{2}$ is $\frac{3}{6}$ and $\frac{1}{3}$ is $\frac{2}{6}$, and $\frac{3}{6}$ and $\frac{2}{6}$ is $\frac{5}{6}$; the boy spent $\frac{5}{6}$ of his money.

Page 106. (96.)

5. $\frac{1}{2}$ is $\frac{5}{10}$ and $\frac{3}{5}$ is $\frac{6}{10}$; $\frac{5}{10}$ and $\frac{6}{10}$ is $\frac{11}{10}$; $\frac{1}{2}$ and $\frac{3}{5}$ is $\frac{11}{10}$.

Page 108. (98.)

7. $\frac{9}{10}$ is $\frac{36}{40}$ and $\frac{3}{4}$ is $\frac{30}{40}$; $\frac{36}{40}$ less $\frac{30}{40}$ is $\frac{6}{40}$, or $\frac{3}{20}$; $\frac{9}{10}$ less $\frac{3}{4}$ is $\frac{3}{20}$.

Page 109. (100.)

2. If John gave $\frac{3}{8}$ of a dollar for a knife and $\frac{1}{4}$, or $\frac{2}{8}$, of a dollar for a ball, for both he gave $\frac{3}{8}$ plus $\frac{2}{8}$, or $\frac{5}{8}$, of a dollar; if he sold both for $\frac{7}{8}$ of a dollar, he gained $\frac{7}{8}$ of a dollar less $\frac{5}{8}$ of a dollar, which is $\frac{2}{8}$, or $\frac{1}{4}$, of a dollar.

3. Jane gave away $\frac{1}{3}$ of a quart plus $\frac{1}{4}$ of a quart; $\frac{1}{3}$ is $\frac{4}{12}$

and $\frac{1}{4}$ is $\frac{3}{12}$, and $\frac{4}{12}$ plus $\frac{3}{12}$ is $\frac{7}{12}$; she gave away $\frac{7}{12}$ of a quart, and she had left $\frac{5}{6}$, or $\frac{10}{12}$, of a quart less $\frac{7}{12}$ of a quart, which is $\frac{3}{12}$, or $\frac{1}{4}$, of a quart.

Page 110. (100.)

5. He spends $\frac{2}{5}$ plus $\frac{1}{10}$ plus $\frac{1}{3}$ of his time; $\frac{2}{5}$ is $\frac{12}{30}$, $\frac{1}{10}$ is $\frac{3}{30}$, and $\frac{1}{3}$ is $\frac{10}{30}$; $\frac{12}{30}$ plus $\frac{3}{30}$ plus $\frac{10}{30}$ is $\frac{25}{30}$, or $\frac{5}{6}$; he spends $\frac{5}{6}$ of his time, and he has left $\frac{6}{6}$ less $\frac{5}{6}$, or $\frac{1}{6}$, of his time.

6. $\frac{1}{6}$ plus $\frac{2}{5}$ of the pole is in the ground and water; $\frac{1}{6}$ is $\frac{5}{30}$ and $\frac{2}{5}$ is $\frac{12}{30}$, and $\frac{5}{30}$ plus $\frac{12}{30}$ is $\frac{17}{30}$, the part of the pole in ground and water; if $\frac{17}{30}$ of it is in the ground and water, and the rest of it in the air, there must be $\frac{30}{30}$ less $\frac{17}{30}$, or $\frac{13}{30}$, of the pole in the air.

7. If he bequeathed $\frac{1}{4}$ of the estate to each of his two sons, both received $\frac{2}{4}$, or $\frac{1}{2}$, of the estate; if the wife received $\frac{1}{3}$, and the two sons $\frac{1}{2}$ of the estate, the wife and sons together received $\frac{1}{3}$ plus $\frac{1}{2}$; $\frac{1}{3}$ is $\frac{2}{6}$ and $\frac{1}{2}$ is $\frac{3}{6}$, and $\frac{2}{6}$ plus $\frac{3}{6}$ is $\frac{5}{6}$; the wife and sons received $\frac{5}{6}$ of the estate, and the daughter received $\frac{6}{6}$ less $\frac{5}{6}$, or $\frac{1}{6}$, of it.

Page 118. (107.)

3. If 5 pounds of cheese cost $\frac{3}{4}$ of a dollar, 1 pound will cost $\frac{1}{5}$ of $\frac{3}{4}$ of a dollar, which is $\frac{3}{20}$ of a dollar.

5. If 6 men can build a wall in $\frac{7}{8}$ of a day, 1 man can build $\frac{1}{6}$ of $\frac{7}{8}$, or $\frac{7}{48}$, of it in a day.

7. If 16½ pounds of sugar are put into 5 parcels, each parcel will contain $\frac{1}{5}$ of 16½ pounds; $\frac{1}{5}$ of 16½ is 3, with 1½ undivided; 1½ is $\frac{3}{2}$, and $\frac{1}{5}$ of $\frac{3}{2}$ is $\frac{3}{10}$; $\frac{3}{10}$ added to 3 gives $3\frac{3}{10}$; hence $3\frac{3}{10}$ pounds were put into each parcel.

Page 120. (110.)

4. $\frac{2}{5}$ is contained in $\frac{4}{5}$ as many times as 2 is contained in 4, which is 2 times.

5. If a pound of butter cost $\frac{1}{4}$ of a dollar, as many pounds can be bought for $\frac{6}{8}$ of a dollar as $\frac{1}{4}$, or $\frac{2}{8}$, is contained times in $\frac{6}{8}$, which is 3; 3 pounds can be bought for $\frac{6}{8}$ of a dollar.

Page 122. (111.)

12. If a man spent $\frac{2}{5}$ of his money, $\frac{3}{5}$ of it was left; if $\frac{3}{5}$ of it was $21, $\frac{1}{5}$ was $\frac{1}{3}$ of $21, or $7, and $\frac{5}{5}$, or the whole, was 5 times $7, or $35; he had $35 at first.

15. $\frac{1}{4}$ more than the cost of the cow is $\frac{5}{4}$ of the cost; if $45 is $\frac{5}{4}$ of the cost, $\frac{1}{4}$ is $9, and $\frac{4}{4}$, or the cost, is $36.

Page 123. (112.)

15. $\frac{2}{5}$ of $\frac{5}{8} = \frac{2}{5} \times \frac{5}{8} = \frac{1}{4}$, part of the factory sold;
$\frac{5}{8} - \frac{1}{4} = \frac{5}{8} - \frac{2}{8} = \frac{3}{8}$, part then owned.

20. $\frac{2}{5}$ of $\frac{5}{7} = \frac{2}{7}$; $\$2400 \div \frac{2}{7} = \$8400$.

22. $\frac{2}{3}$ of $\frac{5}{8} = \frac{5}{12}$; $\$4400 \div \frac{5}{12} = \$10560$.

Page 124. (113.)

23. $\$9760 \times \frac{3}{5} = \$5856$; $\$5856 \div \frac{6}{7} = \$6832$.

Page 144. (146.)

17. Since 1 peck is 8 quarts, $3\frac{3}{4}$ pecks are $3\frac{3}{4}$ times 8 quarts, or 30 quarts; if he sold 30 quarts, at 10 cents a quart, he received 30 times 10 cents, which is 300 cents, or $3.

Page 144. (146.) (Continued.)

20. If he sold $\frac{1}{2}$ bushel, or 16 quarts, of chestnuts, at 8 cents a quart, he received 16 times 8 cents, which is 128 cents, and he made the difference between 128 cents and 100 cents, which is 28 cents.

Page 152. (153.)

31. $\frac{1}{4}$ mile $= \frac{1}{4}$ of 320 rods $=$ 80 rods, length of field;
$\frac{1}{8}$ mile $= \frac{1}{8}$ of 320 rods $=$ 40 rods, width of field;
$80 \times 2 + 40 \times 2 = 240$, rods of fence.

Page 155. (157.)

30. $8 \times 4 = 32$, number of sq. in. in a brick;
$60 \times 12\frac{3}{4} = 765$, sq. ft. in walk;
$765 \times 144 = 110160$, sq. in. in walk;
$110160 \div 32 = 3442\frac{1}{2}$, number of bricks.

31. $15 \times 12 = 180$, sq. ft. in ceiling;
$15 \times 9 \times 2 = 270$ sq. ft. in two side walls;
$12 \times 9 \times 2 = \underline{216}$ sq. ft. in two end walls;
666 sq. ft. in walls and ceiling;
$666 \div 9 = 74$, sq. yd.; $\$.50 \times 74 = \$37.00$, cost.

Page 175.

11. Since a gallon is 4 quarts, or 8 pints, or 16 half-pints, a gallon of sweet oil will fill 16 half-pint bottles.

13. Since every fourth year is leap-year, there are as many leap-years in a century as 4 is contained times in 100, which is 25; there are 25 leap-years in a century.

Page 176.

26. $18\frac{1}{3} \times 15 \div 9 = 30\frac{5}{9}$, sq. yd. in floor.

Since a yard of carpeting one yard wide contains a square yard, it will take $30\frac{5}{9}$ yards of it to carpet the room.

Page 177. (176.)

33. $30 \times 24 \times 60 \times 60 = 2592000$, seconds in June.

Since the clock ticks once a second, it will tick as many times in the month of June as there are seconds, or 2592000 times.

Page 191. (189.)

18. $20 \times 28 \div 9 = 62\frac{2}{9}$, sq. yd. in room;
$62\frac{2}{9} \div \frac{3}{4} = 82\frac{26}{27}$, yards of carpeting.

Since a yard of carpeting $\frac{3}{4}$ of a yard wide contains $\frac{3}{4}$ of a square yard, it will take as many yards of the carpeting to cover the room as $\frac{3}{4}$ sq. yd. is contained times in $62\frac{2}{9}$ sq. yd., which is $82\frac{26}{27}$.

Page 207. (128.)

2. $\frac{8}{1250} = 8.0000 \div 1250 = .0064$.

9. $27.5 \div .025 = 1100$; $76.8 \div .48 = 160$; $1100 \times 160 = 176000$.

COMPLETE ARITHMETIC.

ILLUSTRATIVE SOLUTIONS.

FRACTIONS.

Page 57.

46. $\frac{8}{8} - \frac{5}{8} = \frac{3}{8}$, part standing;
$\frac{3}{5}$ of $\frac{3}{8} = \frac{9}{40}$, part above ground;
$\frac{3}{8} - \frac{9}{40} = \frac{15}{40} - \frac{9}{40} = \frac{6}{40} = \frac{3}{20}$, part in ground.

Page 66.

77. $\frac{3}{8} + \frac{3}{10} = \frac{27}{40}$; $\frac{3}{8} - \frac{3}{10} = \frac{3}{40}$; $\frac{27}{40} \div \frac{3}{40} = 9$.

79. $\frac{7}{8} \div \frac{3}{4} = \frac{7}{6}$; $\frac{5}{9} \div \frac{1}{6} = \frac{10}{3}$; $\frac{7}{6} \times \frac{10}{3} = 3\frac{8}{9}$.

Page 69.

22. If 9 years, or Harry's age, is $\frac{3}{4}$ of Charles's age, $\frac{1}{4}$ of Charles's age is $\frac{1}{3}$ of 9 years, or 3 years; and $\frac{4}{4}$, or his age, is 4 times 3 years, or 12 years. Samuel's age, being $\frac{3}{5}$ of Charles's, is $\frac{3}{5}$ of 12 years, which is $7\frac{1}{5}$ years; Charles is 12 years old and Samuel $7\frac{1}{5}$ years old.

Page 69. (Continued.)

26. Since $\frac{5}{9}$ of the cost of the bridle was $\frac{2}{7}$ of \$35, or \$10, $\frac{1}{9}$ of the cost of the bridle was $\frac{1}{5}$ of \$10, or \$2; and $\frac{9}{9}$, or the whole cost, was 9 times \$2,or \$18.

27. If 15 years added to $\frac{5}{6}$ of a man's age gives $\frac{5}{4}$ of his age, 15 years must equal the difference between $\frac{5}{4}$ and $\frac{5}{6}$ of his age, which is $\frac{5}{12}$ of his age: if $\frac{5}{12}$ of his age is 15 years, $\frac{1}{12}$ of his age is 3 years, and $\frac{12}{12}$, or his age, is 12 times 3 years, or 36 years.

28. $\frac{1}{23}$ of 138 miles is 6 miles, and $\frac{15}{23}$ is 15 times 6 miles, or 90 miles, which is $\frac{3}{4}$ of the distance from Columbus to Cincinnati: if $\frac{3}{4}$ of the distance is 90 miles, $\frac{1}{4}$ is 30 miles and $\frac{4}{4}$, or the whole distance, is 120 miles.

30. If $\frac{2}{3}$ of the value of the house equals $\frac{4}{5}$ of the value of the lot, $\frac{1}{3}$ of the value of the house equals $\frac{2}{5}$ of the value of the lot, and $\frac{3}{3}$, or the whole value of the house, equals $\frac{6}{5}$ of the value of the lot: hence $\frac{6}{5}$ plus $\frac{5}{5}$, or $\frac{11}{5}$, of the value of the lot is \$4400; then $\frac{1}{5}$ is \$400, and $\frac{5}{5}$, or the value of the lot, is \$2000; and $\frac{6}{5}$, or the value of the house, is \$2400.

Page 70.

40. If a train of cars runs $\frac{3}{5}$ of a mile in $\frac{9}{7}$ of a minute, it will run $\frac{1}{5}$ of a mile in $\frac{3}{7}$ of a minute, and $\frac{5}{5}$, or a mile, in $\frac{15}{7}$ of a minute; and it will run as many miles in 15 minutes as $\frac{15}{7}$ is contained times in 15, which is 7.

43. If a rod 5 feet long casts a shadow $8\frac{1}{3}$ feet long, each foot of the rod casts a shadow $\frac{1}{5}$ of $8\frac{1}{3}$ feet, which is $\frac{5}{3}$ of a foot, and if a pole, at the same time of day, casts a shadow $17\frac{1}{2}$ feet long, the pole must be as many feet

Page 70. (Continued.)

long as $\frac{5}{3}$ of a foot is contained times in $17\frac{1}{2}$ feet, which is $10\frac{1}{2}$; or,

Dividing 5 feet by $8\frac{1}{3}$ feet, we find that the rod is $\frac{3}{5}$ of the length of its shadow, and, at the same time of day, the pole must be $\frac{3}{5}$ of the length of its shadow, or $\frac{3}{5}$ of $17\frac{1}{2}$ feet, which is $10\frac{1}{2}$ feet.

44. It will take 1 man 3 times as long as 3 men to do the work, or 3 times $10\frac{3}{4}$ days, which is $32\frac{1}{4}$ days; and it will take 8 men $\frac{1}{8}$ of the time of 1 man, or $\frac{1}{8}$ of $32\frac{1}{4}$ days, which is $4\frac{1}{32}$ days.

47. If A and B can do the work in 8 days, they can do $\frac{1}{8}$ of it in one day, and if A can do it in 12 days, he can do $\frac{1}{12}$ of it in one day, and the difference between $\frac{1}{8}$ and $\frac{1}{12}$, or $\frac{1}{24}$, is the part of the work which B can do in one day: if B can do $\frac{1}{24}$ of the work in one day, it will take him as many days to do the work as $\frac{1}{24}$ is contained times in $\frac{24}{24}$, which is 24.

48. If A and B can mow a field in 10 days, they can mow $\frac{1}{10}$ of it in one day, and if A can mow only $\frac{2}{3}$ as much as B, A and B together can mow $\frac{2}{3}$ plus $\frac{3}{3}$, or $\frac{5}{3}$, as much as B, and hence $\frac{1}{10}$ of the field is $\frac{5}{3}$ of what B can mow in one day: if $\frac{1}{10}$ is $\frac{5}{3}$ of what B can mow in one day, $\frac{1}{50}$ is $\frac{1}{3}$, and $\frac{3}{50}$ is all that he can mow in one day; and if B can mow $\frac{3}{50}$ of the field in one day, it will take him as many days to mow the whole field as $\frac{3}{50}$ is contained times in $\frac{50}{50}$, which is $16\frac{2}{3}$. If A and B together can mow $\frac{1}{10}$ of the field in one day, and B can mow $\frac{3}{50}$ of it, the difference between $\frac{1}{10}$ and $\frac{3}{50}$, or $\frac{1}{25}$, is what A can mow in one day, and if A can mow $\frac{1}{25}$ of the field in one day, he can mow all of it in 25 days.

Page 70. (Continued.)

49. Adding 5 to each term of the proper fraction $\frac{2}{3}$ gives $\frac{7}{8}$, and reducing to a common denominator shows that $\frac{7}{8}$ is greater than $\frac{2}{3}$. Subtracting 7 from each term of the proper fraction $\frac{11}{12}$ gives $\frac{4}{5}$, and reducing to a common denominator, as before, shows that $\frac{4}{5}$ is less than $\frac{11}{12}$. From these examples it would seem that, adding the same number to both terms *increases*, and subtracting the same number from both terms *decreases*, the value of a proper fraction. This will be found to be true in any case that may be tried, and the general principle involved may be proved by means of algebra.

Page 71.

50. Adding 4 to each term of the improper fraction $\frac{7}{6}$ (greater than 1) gives $\frac{11}{10}$, and reducing to a common denominator shows that $\frac{11}{10}$ is less than $\frac{7}{6}$. Subtracting 4 from each term of $\frac{7}{6}$ gives $\frac{3}{2}$, and reducing to a common denominator shows that $\frac{3}{2}$ is greater than $\frac{7}{6}$. From these examples it would seem that adding the same number to both terms *decreases*, and subtracting the same number from both terms *increases*, the value of an improper fraction which is greater than unity. The same will be found to be true in any case that may be tried, as can be proved by algebra.

[The pupil will notice that if the same number be added to, or subtracted from, both terms of an improper fraction which is *equal* to unity, the resulting fraction will equal unity, there being no change in value.]

63. \$68 $\div \frac{4}{15} =$ \$255, cost of 1 acre;
\$255 $\times 12\frac{2}{7} =$ \$3132$\frac{6}{7}$, cost of 12$\frac{2}{7}$ acres.
Or, \$68 $\times \frac{15}{4} \times \frac{86}{7} =$ \$3132$\frac{6}{7}$.

Page 71. (Continued.)

65. $\frac{5}{7}\times\frac{12}{25}=\frac{12}{35}$; $\frac{35}{35}-\frac{12}{35}=\frac{23}{35}$;
$69\div\frac{23}{35}=105$.

Page 72.

69. $\frac{15}{15}-\frac{7}{15}=\frac{8}{15}$; $208\div\frac{8}{15}=390$.

If $\frac{7}{15}$ of the pupils are girls, $\frac{15}{15}-\frac{7}{15}$, or $\frac{8}{15}$, are boys; hence 208 is $\frac{8}{15}$ of the number of pupils enrolled in the school.

70. $\frac{5}{9}\times\frac{3}{7}=\frac{5}{21}$; $\$3480\div\frac{5}{21}=\$14616$.

71. $\frac{3}{5}\times\frac{2}{5}=\frac{6}{25}$; $\frac{3}{5}-\frac{6}{25}=\frac{9}{25}$; $\frac{9}{25}\times\frac{1}{2}=\frac{9}{50}$, C's part;
$\$460\div\frac{9}{50}=\$2555\frac{5}{9}$.

72. $\frac{7}{12}+\frac{7}{16}=\frac{28}{48}+\frac{21}{48}=\frac{49}{48}$; $\frac{49}{48}\times\frac{3}{10}=\frac{49}{160}$, C's part.

73. $\frac{2}{3}\times\frac{3}{7}=\frac{2}{7}$, sold to B; $\frac{2}{3}\times\frac{2}{5}=\frac{4}{15}$, sold to C; $\frac{2}{7}+\frac{4}{15}=\frac{30}{105}+\frac{28}{105}=\frac{58}{105}$, sold to B and C; $\frac{2}{3}-\frac{58}{105}=\frac{70}{105}-\frac{58}{105}=\frac{12}{105}=\frac{4}{35}$, A's remaining share.

74. If $\frac{3}{5}$ A's $=\frac{3}{4}$ B's, then $\frac{1}{5}$ A's $=\frac{1}{4}$ B's, and $\frac{5}{5}$, or the whole of A's, $=\frac{5}{4}$ B's: if A's $=\frac{5}{4}$ of B's, A's and B's together, or $\frac{9}{4}$ of B's, $=396$ acres; $396\div\frac{9}{4}=176$, B's; $176\times\frac{5}{4}=220$, A's.

75. $\frac{3}{8}+\frac{5}{11}=\frac{33}{88}+\frac{40}{88}=\frac{73}{88}$; $\frac{88}{88}-\frac{73}{88}=\frac{15}{88}$, C's part.
$\$6160\times\frac{3}{8}=\$2310$, A's share;
$\$6160\times\frac{5}{11}=\$2800$, B's share;
$\$6160\times\frac{15}{88}=\$1050$, C's share.

76. $\frac{8}{8}-\frac{5}{8}=\frac{3}{8}$; $\frac{3}{8}\times\frac{3}{5}=\frac{9}{40}$, part damaged by water;
$\frac{3}{8}-\frac{9}{40}=\frac{6}{40}=\frac{3}{20}$, part sold for \$5280;
$\$5280\div\frac{3}{20}=\$35200$.

Page 72. (Continued.)

77. $\frac{5}{5}-\frac{2}{5}=\frac{3}{5}$; $\frac{3}{5}\times\frac{1}{3}=\frac{1}{5}$, part paid for repairs;
$\frac{3}{5}-\frac{1}{5}=\frac{2}{5}$; $\frac{2}{5}\times\frac{1}{3}=\frac{2}{15}$, part paid for stock;
$\frac{2}{5}-\frac{2}{15}=\frac{4}{15}$; $\frac{4}{15}\times\frac{1}{2}=\frac{2}{15}$, part paid for utensils;
$\frac{4}{15}-\frac{2}{15}=\frac{2}{15}$, what remained;
\$650 $\div\frac{2}{15}=$ \$4875, amount of money.
Or, $\frac{3}{5}\times\frac{2}{3}\times\frac{2}{3}\times\frac{1}{2}=\frac{2}{15}$; \$650 $\div\frac{2}{15}=$ \$4875.

N. B.—The successive fractional factors denote the fractional part *left* after each expenditure.

78. $3\frac{3}{4}+2\frac{7}{8}+1\frac{4}{5}=8\frac{17}{40}$, number of yards in one suit, consisting of coat, pants, and vest.

$67\frac{2}{5}$ yds. $\div\ 8\frac{17}{40}$ yds. $=8$, number of each, or number of suits.

79. $\frac{3}{8}+\frac{3}{10}=\frac{27}{40}$; $\frac{40}{40}-\frac{27}{40}=\frac{13}{40}$, sister's share;
$\frac{3}{8}-\frac{13}{40}=\frac{1}{20}$, difference between elder brother's share and sister's share;
\$450 $\div\frac{1}{20}=$ \$450 $\times 20=$ \$9000, entire estate;
\$9000 $\times\frac{3}{8}=$ \$3375, elder brother's share;
\$9000 $\times\frac{3}{10}=$ \$2700, younger brother's share.

MENSURATION.

Page 103.

10. The circumference of any circle is nearly 3.1416 times its diameter (Art. 157); since the diameter of the given circle is 10 feet, the circumference is 10 feet $\times$ 3.1416 $=$ 31.416 feet.

Page 104.

18. $15 \times 8\frac{1}{4} = 123\frac{3}{4}$, square yards in room;
$123\frac{3}{4} \div \frac{3}{4} = 165$, yards of carpeting required.

Since the carpeting is $\frac{3}{4}$ of a yard wide, a yard of it will contain $\frac{3}{4}$ of a *square yard*, and hence it will take as many yards of the carpeting to cover the room as $\frac{3}{4}$ sq. yd. is contained times in $123\frac{3}{4}$ sq. yd., which is 165.

Or thus: It is required to find the length of a rectangle (strip of carpet), having an area of $123\frac{3}{4}$ square yards, and being $\frac{3}{4}$ of a yard wide. We have given a product ($123\frac{3}{4}$) and one factor ($\frac{3}{4}$) to find the other.

20. $270 \div 36 = 7\frac{1}{2}$; $7\frac{1}{2} \times 2 = 15$.

Since the area of a triangle is the product of the base by half the altitude, the area, 270, divided by the base, 36, will give half the altitude, $7\frac{1}{2}$, and hence the altitude is $7\frac{1}{2}$ in. × 2 = 15 in.

21. 12 in. × 3.1416 = 37.6992 in., circumference (Art. 157);
37.6992 in. $\times \frac{12}{4} = 113.0976$ sq. in. (Art. 161, 4.)

Page 106.

12. $65 \times 12 \times 2 \times 2 = 3120$, cu. ft. in two side walls;
$40 \times 12 \times 2 \times 2 = 1920$, cu. ft. in two end walls;
$\overline{5040}$, cu. ft. in all the walls.
Or, $65 + 65 + 40 + 40 = 210$, length of four walls;
$210 \times 12 \times 2 = 5040$, cu. ft. in four walls.

If the side walls are considered 65 feet long, the two end walls must each be considered 4 feet less than the width of the building, since the two side walls are each 2 feet thick, and thus take 2 feet from each end of the end walls.

Page 106. (Continued.)

NOTE.—In measuring the walls of cellars and buildings, masons take the distance around the *outside* of the walls (the girth) for the length, thus measuring each corner *twice.* See Comp. Arith., p. 118, Note. But in finding the exact number of cubic feet in the walls of a building, as is required in the above problem, each corner should be counted but once. The answer to this problem in the earlier editions of the arithmetic was incorrect. The correct answer is 5040 cubic feet.

15. $6 \times 4 \times 2 = 48$, cu. inches in a can.
$30 \times 18 \times 20 = 10800$, cu. inches in box.
$10800 \div 48 = 225$, number of cans.

N. B.—This solution would not be correct if the dimensions of the box were not multiples of the dimensions of the can, thus permitting the cans to be packed *without loss of space.*

16. $12 \times 3.1416 = 37.6992$, inches in circumference of base (Art. 157);
$37.6992 \times \frac{12}{4} = 113.0976$, sq. inches in base (Art. 164, 4);
$113.0976 \times 25 = 2827.44$, cu. inches in cylinder (Art. 167, 3).

DENOMINATE NUMBERS.

Page 112.

49. $.012 \times 8 \times 40 \times 5\frac{1}{2} = 21.12$, yards.

50. $\frac{4}{5} = .8$; $.8 \div 5.5 \div 40 \div 8 = .00045\frac{5}{11}$, miles; or, $\frac{4}{5} \div 5\frac{1}{2} = \frac{8}{55}$; $\frac{8}{55} \div 8 = \frac{1}{55}$; $\frac{1}{55} \div 40 = \frac{1}{2200} = .00045\frac{5}{11}$.

Page 115.

20. 1 mile $= 1760$ yards.
1 fur. 18 rd. 1 yd. $= 320$ yd.
$320 \div 1760 = .18\frac{2}{11}$.

24. 3 barrels $= 3 \times 196 \times 16$ oz. $= 9408$ oz.
110 lb. 4 oz. $= 1764$ oz.
$1764 \div 9408 = \frac{3}{16}$.

Page 117.

14. $60 \times 2 + 37\frac{1}{2} \times 2 = 195$, rods, distance around the field.
$195 \times 16\frac{1}{2} = 3217\frac{1}{2}$, feet.
$3217\frac{1}{2} \div 12 = 268\frac{1}{8}$, number of boards to go around once.
$268\frac{1}{8} \times 5 = 1340\frac{5}{8}$, number of boards to make fence 5 boards high.

NOTE.—In practice it would require 1345 boards to make the fence. It would require 825 boards to make the two side fences, and since each end is $51\frac{9}{16}$ times 12 feet, it will take 5 times 52 boards, or 260 boards, to make the fence across each end, and 520 boards for both ends.

Page 118.

19. $45 \times 2 + 34 \times 2 = 158$ ft., length of walls measuring around the outside;
$158 \times 8 \times 2\frac{1}{6} = 2738\frac{2}{3}$, cu. ft. in wall;
$2738\frac{2}{3} \div 24\frac{3}{4} = 110 +$, perches in wall.

NOTE.—Masons defend their practice of measuring each corner twice with the plea that the work of *turning a corner* is equivalent to laying double the amount of masonry.

20. $120 \times 4 = 480$, ft., outside girth;
$480 \times 33\frac{1}{3} \times 11 = 176000$, cu. ft. in walls;
$176000 \div 24\frac{3}{4} = 7111\frac{1}{9}$, perches in walls.

Page 118. (Continued.)

23. 165 ÷ $5\frac{1}{2}$ = 30, rods in diameter;
30 × 3.1416 = 94.248, rods in circumference;
94.248 × $\frac{30}{4}$ = 706.86, sq. rods in park;
706.86 ÷ 160 = 4.417875, acres in park.

24. $3\frac{1}{3}$ × 3.1416 = 10.472, ft. in circumference;
10.472 × $\frac{1}{4}$ of $3\frac{1}{3}$ = $8.72\frac{2}{3}$, sq. ft., area of bottom;
$8.72\frac{2}{3}$ × 20 = $174.5\frac{1}{3}$, cu. ft. in well.

25. 5 × 3.1416 = 15.708, ft. in circumference;
15.708 × $\frac{5}{4}$ = 19.635, sq. ft., area of bottom;
19.635 × $6\frac{1}{3}$ = 124.355, cu. ft. in cistern;
124.355 × 1728 = 214885.44, cu. inches in cistern;
214885.44 ÷ 231 = 930.24, gallons in cistern.

Page 124.

34. 2.8375 bu. × 250 = 709.375 bu.

36. 2.2046 lb. × 150 = 330.69 pounds.

37. .0164 liter × 1728 × 35 = 991.872 liters.

39. .4047 hectares × $\frac{3}{4}$ = .30352 hectares;
.30352 × 100 = 30.352, ares.

40. .3048 meters × 1760 = 536.448 meters.

COMPOUND NUMBERS.

Page 126.

17. 37° 20′ N. + 12° 36′ N. = 49° 56′ N.;
15° 45′ S. + 18° 40′ S. = 34° 25′ S.;
49° 56′ N. — 34° 25′ S. = 15° 31′ N.

Page 129.

16. $8 \times 40 \times 16\frac{1}{2} \times 12 \times \frac{5}{8} = 39600$, inches in $\frac{5}{8}$ mile;
12 ft. 6 in. $= 150$ in.; $39600 \div 150 = 264$.

17. 20 rd. 5 yd. $= 4140$ in.; 11 ft. 6 in. $= 138$ in.;
4140 in. $\div$ 138 in. $= 30$, lengths on each side;
$30 \times 4 = 120$, lengths of fence on four sides.

20. $45 \times 4 \times 16\frac{1}{2} \times 12 = 35640$, in. in four sides of field;
$35640 \div 30 = 1188$, steps.

Page 133.

23. $11.45 - 9.30 = 2.15 = 2\frac{1}{4}$, hours;
$2\frac{1}{4} \times 15 = 33\frac{3}{4} = 33° \ 45'$, east of New York;
$74° - 33° \ 45' = 40° \ 15'$, W., long. of vessel.

The fact that the time on the ship was *later* than the time at New York, at the moment of the eclipse, shows that the ship was *east* of New York.

24.

$$\begin{array}{lll} & 45 \text{ min.} & 30 \text{ sec.} \\ & & 15 \\ \hline 11° & 22' & 30'' \\ 85° & 40' & \\ \hline 97° & 2' & 30'' \end{array}$$

It is seen at once that the observatory having the faster time, is east of the other.

25. $843 \div 51 = \frac{281}{17}°$; $\frac{281}{17}° \div 15 = \frac{281}{255}$ hr. $= 1$ hr. $6\frac{2}{17}$ min.; 12 hr. $-$ 1 hr. $6\frac{2}{17}$ min. $= 10$ hr. $53\frac{15}{17}$ min.

NOTE. — In the earlier editions of the Complete Arithmetic, the length of a degree of longitude at Boston was

Page 133. (Continued.)

given as about 52 miles, which made the distance from Boston to Chicago about 890 miles, and the difference in time about 1 hr. $8\frac{6}{13}$ min.

$890 \div 52 = \frac{445}{26}°$; $\frac{445}{26}° \div 15 = \frac{89}{78}$ hr.

26. $714 \div 54 = \frac{714}{54}°$; $\frac{714}{54}° \div 15 = \frac{714}{810}$ hr.;
$\frac{714}{810} \times 60 = 52\frac{8}{9}$ min.; 9 hr. + $52\frac{8}{9}$ min.

NOTE. — In the earlier editions the length of a degree of longitude at Washington was given as about 53 miles, which made the distance between Washington and Chicago about 690 miles, and the difference in time about $52\frac{4}{53}$ minutes.

Page 134.

28. The longitude of Cincinnati (84° 26′ W.) is given in problem 17, page 133.

29. The longitude of Boston (71° 4′ 9″ W.) is given in problem 19, page 133.

30. The longitude of San Francisco (122° 26′ 15″ W.) is given in problem 17, page 133.

31. The longitude of New York (74° W.) is given in problem 23, page 133.

32. The longitude of Berlin (13° 23′ 53″ E.) is given in problem 20, page 133.

NOTE. — In the later editions of the Complete Arithmetic the longitude of the above cities is also given in problems 28, 29, 30, 31, and 32, respectively.

PERCENTAGE.

Page 138.

16. $\$540 \times .00\frac{1}{2} = \$2.70.$
Or, $\$5.40 = 1\%;\ \$5.40 \times \frac{1}{2} = \$2.70.$

17. $\$4000. \times .00\frac{1}{10} = \$4.$
Or, $\$40. = 1\%;\ \$40. \times \frac{1}{10} = \$4.$

18. 21700 feet $\times .00\frac{3}{7} = 93$ ft.
Or, 217 feet $= 1\%$; 217 feet $\times \frac{3}{7} = 93$ ft.

19. $\$48.50 \times .00\frac{3}{4} = \$.36375.$
Or, $\$.485 = 1\%;\ \$.485 \times \frac{3}{4} = \$.36375.$

20. 965 days $\times .33\frac{1}{3} = 321\frac{2}{3}$ days.
Or, $33\frac{1}{3}\% = \dfrac{33\frac{1}{3}}{100} = \frac{1}{3}$; 965 days $\times \frac{1}{3} = 321\frac{2}{3}$ days.

21. $16\frac{2}{3}\% = \dfrac{16\frac{2}{3}}{100} = \frac{1}{6};\ \$.54 \times \frac{1}{6} = \$.09.$

22. $\frac{5}{8} \times \frac{15}{100} = \frac{3}{32}.$

23. $66\frac{2}{3}\% = \dfrac{66\frac{2}{3}}{100} = \frac{2}{3};\ \frac{7}{15} \times \frac{2}{3} = \frac{14}{45}.$

26. $33\% - 25\frac{1}{2}\% = 7\frac{1}{2}\%$; 480 m. $\times .07\frac{1}{2} = 36$ miles.
Or, $480 \times .33 = 158.4;\ 480 \times 25\frac{1}{2} = 122.4;$
$158.4 - 122.4 = 36$, miles.

30. $58500 \times .37\frac{1}{2} = 21937.5$, bushels by steamer;
$58500 \times .33\frac{1}{3} = 19500$, bushels by schooner;
$21937.5 + 19500 = 41437.5;$
$58500 - 41437.5 = 17062.5$, bushels by railroad.

Page 139.

20. $\frac{1}{3} \div \frac{5}{6} = \frac{6}{15} = \frac{2}{5} = .40 = 40\%$.

We first find that $\frac{1}{3}$ is $\frac{2}{5}$ of $\frac{5}{6}$, and by changing $\frac{2}{5}$ to a decimal we find that $\frac{1}{3}$ is .40 or 40% of $\frac{5}{6}$.

21. $\frac{2}{3} \div \frac{4}{5} = \frac{10}{12} = \frac{5}{6} = .83\frac{1}{3} = 83\frac{1}{3}\%$.

22. $\frac{3}{4} \div \frac{5}{2} = \frac{3}{10} = .30 = 30\%$.

24. $\frac{22}{24} = .91\frac{2}{3} = 91\frac{2}{3}\%$.

An alloy of gold is said to be as many carats fine as it contains twenty-fourths of pure gold; thus, in the present example, $\frac{22}{24}$ of the ring is pure gold.

25. $16\frac{2}{3}\,\% = \frac{16\frac{2}{3}}{100} = \frac{1}{6}$; $\$150 \times \frac{1}{6} = \$25$;
$\$25 \div \$45 = .55\frac{5}{9} = 55\frac{5}{9}\%$.

Page 140.

28. (1) $\frac{5}{8} = .62\frac{1}{2} = 62\frac{1}{2}\%$; (2) $\frac{3}{25} = .12 = 12\%$.

Page 141.

17. $60\% + 25\% = 85\%$; $100\% - 85\% = 15\%$;
$\$7.50 \div 15 = \$\ .50$ (1%); $\$.50 \times 100 = \$50$.
Or, $\$7.50 \div .15 = \$50$.

20. $60\% = .60$; $40\% = .40$; $.60 \times .40 = .24$;
$\$9600 \div .24 = \$40000$.
Or, $60\% = \frac{60}{100} = \frac{3}{5}$; $40\% = \frac{40}{100} = \frac{2}{5}$;
$\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$; $\$9600 \div \frac{6}{25} = \$40000$.

Page 143.

12. $100\% - 31\frac{1}{4}\% = 68\frac{3}{4}\% = .6875$;
$\$400 \div .6875 = \$581.81\frac{9}{11}$.

13. $100\% + 15\% = 115\% = 1.15$;
$276 \div 1.15 = 240$.

14. $100\% + 30\% = 130\% = 1.30$;
$162.5 \div 1.30 = 125$.

15. $100\% - 16\frac{2}{9}\% = 83\frac{7}{9}\% = .83\frac{7}{9}$;
$2035.8 \div .83\frac{7}{9} = 2430$.

16. $100\% - 32\% = 68\% = .68$;
$306 \div .68 = 450$, acres.

17. $100\% + 25\% = 125\% = 1.25$;
$\$150 \div 1.25 = \$120$.
Or, $125\% = \frac{125}{100} = \frac{5}{4}$; if $\frac{5}{4} = \$150$, $\frac{1}{4} = \$30$, and $\frac{4}{4} = \$120$.

18. $100\% + 50\% = 150\% = 1.50$;
$\$1 \div 1.50 = \$\ .66\frac{2}{3}$.

19. $100\% - 5\% = 95\% = .95$;
$570 \div .95 = 600$, pupils.

20. $100\% + 15\% = 115\% = 1.15$;
$920 \div 1.15 = 800$.

21. $100\% + 18\% = 118\% = 1.18$;
$171572 \div 1.18 = 145400$.

Page 144.

1. (1) $12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{1}{8}$; $640 \times \frac{1}{8} = 80$.

2. (1) $33\frac{1}{3}\%$ of $672 = \frac{1}{3}$ of $672 = 224$.

Page 144. (Continued.)

3. (1) 15 is $\frac{1}{4}$ of 60, and $\frac{1}{4}$ is $\frac{25}{100}$ or 25%.

4. (1) $16\frac{2}{3}$ is $\frac{1}{3}$ of 50, and $\frac{1}{3}$ is $33\frac{1}{3}\%$.

5. 25% of 120 is $\frac{1}{4}$ of 120, which is 30, and 30 is $\frac{1}{3}$, or $33\frac{1}{3}\%$, of 90.

6. $33\frac{1}{3}\%$ of 150 is $\frac{1}{3}$ of 150, which is 50, and 50 is $\frac{1}{5}$, or 20%, of 250.

7. $12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{1}{8}$; 80 is $\frac{1}{8}$ of 640.

8. $20\% = \frac{20}{100} = \frac{1}{5}$; $\frac{1}{5}$ of 105 is 21;
$25\% = \frac{25}{100} = \frac{1}{4}$; 21 is $\frac{1}{4}$ of 84.

9. $33\frac{1}{3}\%$ of 225 is $\frac{1}{3}$ of 225, or 75; if 75 is 15%, 5 is 1%, 5 × 100, or 500, is 100%, or the required number.

10. 75% of 120 is $\frac{3}{4}$ of 120, or 90; 45 is $\frac{1}{2}$, or 50%, of 90.

11. If 360 is 20% more than a number, it is $\frac{20}{100}$, or $\frac{1}{5}$ more, and hence is $\frac{6}{5}$ of it; if 360 is $\frac{6}{5}$ of a number, 60 is $\frac{1}{5}$, and 300 is the number.

12. If 60 is $33\frac{1}{3}\%$ more than a number, it is $\frac{1}{3}$ more, and hence is $\frac{4}{3}$ of it; if 60 is $\frac{4}{3}$ of a number, 15 is $\frac{1}{3}$, and 45 is the number.

13. $33\frac{1}{3}\%$ of 240 is $\frac{1}{3}$ of 240, which is 80, and if 80 is $33\frac{1}{3}\%$, or $\frac{1}{3}$ less than a number, it is $\frac{2}{3}$ of it; if 80 is $\frac{2}{3}$ of a number, 40 is $\frac{1}{3}$, and 120 is the number.

14. 25% of 280 is $\frac{1}{4}$ of 280, which is 70, and if 70 is $16\frac{2}{3}\%$, or $\frac{1}{6}$, more than a number, it is $\frac{7}{6}$ of it; if 70 is $\frac{7}{6}$, 10 is $\frac{1}{6}$, and 60 is the number.

Page 144. (Continued.)

15. 20%, or $\frac{1}{5}$, of 60 years is 12 years, and if 12 years are 25%, or $\frac{1}{4}$, of the wife's age, her age is 48 years.

16. $63500 \times .17\frac{1}{5} = 10922$; $63500 + 10922 = 74422$.

Page 145.

17. $480 \times .30 = 144$, meadow;
$480 \times .25\frac{1}{2} = 122.4$, pasture;
$480 \times 16\frac{2}{3} = 80$, grain land;
$144 + 122.4 + 80 = 346.4$;
$480 - 346.4 = 133.6$, woodland.

18. $\$6300 \div \$10500 = .60 = 60\%$.

19. $\$275 + \$180 + \$150 = \$605$, expenses;
$\$950 - \$605 = \$345$, amount left;
$\$345 \div \$950 = .36\frac{6}{19} = 36\frac{6}{19}\%$.

20. $100\% - 35\% = 65\%$, of income, the amount spent;
$\$280 + \$175 = \$455$; $\$455 \div .65 = \$700$.

21. $\$27000 \times .10 = \$2700$; $\$2700 \div .33\frac{1}{3} = \$8100$, income;
$\$8100 \times .80 = \$6480$, expenses.

22. $\$10811 \div 1.13\frac{4}{5} = \$9500$.

23. $\$9450 \div .66\frac{2}{3} = \$14175$.

24. $1200 \times .45 = 540$, apple; $1200 \times .22 = 264$, peach;
$1200 \times .12\frac{1}{2} = 150$, cherry; $540 + 264 + 150 = 954$;
$1200 - 954 = 246$, pear.

25. $\$35000 \times .42\frac{1}{2} = \$14875$, A's; $\$35000 \times .37 = \$12950$, B's; $\$14875 + \$12950 = \$27825$; $\$35000 - \$27825 = \$7175$, C's share.

Page 145. (Continued.)

26. $\$7560 \div .62\frac{1}{2} = \$12096$, children's share;
$\$7560 + \$12096 = \$19656$; $\$19656 \div .80 = \$24570$.

27. $41064 \div 1.16 = 35400$, population in 1860;
$35400 \div .93\frac{3}{4} = 37760$, population in 1865.

28. $1950 \div .03\frac{1}{4} = 60000$.

Page 147.

6. The amount of gain is $1, which is $\frac{1}{4}$, or $\frac{25}{100}$, of the cost price; hence he gains 25%.

7. The amount of loss is $1, which is $\frac{1}{5}$, or $\frac{20}{100}$, of the cost price; hence he loses 20%.

8. He gains 30 cts. a pound on each kind; and 30 cts. is $\frac{1}{4}$, or $\frac{25}{100}$, of $1.20, and $\frac{1}{5}$, or $\frac{20}{100}$, of $1.50; hence he gains 25% on the first and 20% on the second.

12. If he gained 20%, or $\frac{1}{5}$, $90 is $\frac{6}{5}$ of the cost; if $90 is $\frac{6}{5}$, $15 is $\frac{1}{5}$, and $75 is the cost.

13. If he lost 20%, or $\frac{1}{5}$, $80 is $\frac{4}{5}$ of the cost; if $80 is $\frac{4}{5}$, $20 is $\frac{1}{5}$, and $100 is the cost.

14. If 25%, or $\frac{1}{4}$, is gained, 40 cts. is $\frac{5}{4}$ of the cost; if 40 cts. is $\frac{5}{4}$ of cost, 8 cts. is $\frac{1}{4}$, and 32 cts. is the cost.

16. (1) To gain 25%, the selling price must be 25%, or $\frac{1}{4}$, more than the cost; $\frac{5}{4}$ of $2 is $2.50; $\frac{5}{4}$ of $2.50 is $\$3.12\frac{1}{2}$; $\frac{5}{4}$ of $3 is $3.75.

(2) To gain 30%, the selling prices must be 30%, or $\frac{3}{10}$, more than the cost; $\frac{13}{10}$ of $2 is $2.60; $\frac{13}{10}$ of $2.50 is $3.25; $\frac{13}{10}$ of $3 is $3.90.

Page 147. (Continued.)

18. If 20%, or $\frac{1}{5}$, is gained, 90 cts. is $\frac{6}{5}$ of the cost, and if 90 cts. is $\frac{6}{5}$, 75 cts. is the cost; if sold for $1.00, the gain was 25 cents, which is $\frac{1}{3}$, or $33\frac{1}{3}$%, of 75 cents.

Page 148.

24. $130 + $6 = $136, cost; $153.50 — $136 = $17.50, gain;

$17.50 ÷ $136. = .12$\frac{59}{68}$ = 12$\frac{59}{68}$%.

27. $45.10 ÷ .22 = $205, cost; $205 ÷ 82 = $2.50.

28. $180 ÷ .83$\frac{1}{3}$ = $216.

29. $7762.50 ÷ 1.15 = $6750.

30. $45000 × $\frac{2}{5}$ = $18000; $18000 ÷ 1.20 = $15000;
$45000 × $\frac{1}{2}$ = $22500; $22500 ÷ 1.25 = $18000;
$18000 + $22500 = $40500; $45000 — $40500 = $4500;
$4500 ÷ 1.33$\frac{1}{3}$ = $3375;
$15000 + $18000 + $3375 = $36375.

31. $240 ÷ .80 = $300, cost price; $300 × 1.20 = $360.
If 20% is lost at $240, $240 is 80% of cost price.

32. 130% × .88 = 114$\frac{2}{5}$%, the wholesale price;
114$\frac{2}{5}$% — 100% = 14$\frac{2}{5}$%, gain at wholesale.

100% may represent the cost price; then 130% is the retail price, and 12% less than this is the same as 88% of it.

33. $3.50 × 1.15 = $4.025; $4.025 ÷ .85 = $4.735 +
Or, 115% ÷ .85 = 135$\frac{5}{17}$%; $3.50 × 1.35$\frac{5}{17}$ = $4.735 +

The cost may be represented by 100%, the selling price is 15% more, or 115% of the cost, and this is to be 15% *less*, or 85% of the marked price.

Page 149.

34. 100% + 25% = 125%; 125% × .80 = 100%.

25% more than 100%, the cost, is 125%, or the marked price; and 20% less, or 80% of 125%, is 100%; hence he sold at 100% of the cost, and neither gained nor lost.

Page 151.

9. The $108 received was the amount to be expended for peaches plus 8% of that amount, and hence $108 was 108% of the amount paid for peaches, and 1% is $1, and 100% is $100.

10. $309 was 103% of the amount to be expended for flour, and 1% is $3; hence 100% is $300.

11. $7.50 is the remainder after taking away 25% of bill, and hence must be 75% of it; if $7.50 is 75% of bill, 1% is 10 cents, and 100% is $10; $10 — $7.50 = $2.50, commission.

12. The commission being 20%, the proceeds, or $160, were 80% of the sales; if $160 is 80%, 1% is $2 and 100% is $200, amount of sales.

16. $950 × .65 = $617.50, amount collected; $617.50 × .06¼ = $38.593, commission; $617.50 — $38.593 = $578.907, amount paid over.

17. $75000 × .01¾ = $1312.50, commission;
$75000 + $1312.50 = $76312.50.

18. ¼% + 1⅜% = 1⅝%; $120000 × .01⅝ = $1950.

Page 152.

23. $1.75 × 3300 = $5775, cost of apples;
$5775 × .03 = $173.25, commission;
$5775 + $173.25 = $5948.25.

24. $3529.20 ÷ 1.02 = $3460, amount paid for bacon;
$3529.20 — $3460 = $69.20, commission.

25. $11774 ÷ 1.015 = $11600, amount paid for cotton;
$11774 — $11600 = $174, commission.

28. $7.50 × 640 = $4800, proceeds;
$4800 × .01½ = $72, commission;
$4800 — $72 = $4728, amount to be invested in stock and commission;
$4728 ÷ 1.00⅞ = $4686.988;
$4728 — $4686.988 = $41.012, commission on stock;
$72 + $41.012 = $113.012.

29. $1.75 × 750 = $1312.50; $1312.50 × .02¼ = $29.531, commission; $1312.50 + $29.531 = $1342.031; $1342.031 × .00¼ = $3.355, cost of draft;
$1342.031 + $3.355 + $12.50 = $1357.886.

Freight is paid at place of delivery of goods, and hence the $12.50 is not to be included in the draft.

30. $5000 ÷ 1.025 = $4878.0487, amount to be invested in wheat;
$4878.0487 ÷ $1.10 = 4434.59, bushels.

Page 153.

31. $75 × 45 = $3375; $125 × 9 = $1125; $3375 + $1125 = $4500, sales; $4500 — $3375 = $1125, commission; $1125 ÷ $4500 = .25 = 25%, rate of commission.

Page 153. (Continued.)

32. \$15000 × .90 = \$13500, net proceeds;
\$13500 ÷ 1.05 = \$12857.142.

33. \$.22 × 3040 = \$668.80; \$.15 × 10560 = \$1584; \$668.80 + \$1584 = \$2252.80; \$2252.80 × .95 = \$2140.16; \$2140.16 ÷ 1.03 = \$2077.825, amount invested; \$2252.80 — \$2077.825 = \$174.975, commission.

34. \$5.75 × 1300 = \$7475; \$7475 × .96½ = \$7213.375, net proceeds;
\$7213.375 ÷ 1.02 = \$7071.936, amount invested;
\$7071.936 ÷ .28 = 25256.914, pounds;
\$7475 — \$7071.936 = \$403.064, commission.

Page 157.

14. \$5200 × 1.27 = \$6604, amount paid; \$3600 × 1.35 = \$4860; \$1600 × 1.37½ = \$2200; \$4860 + \$2200 = \$7060, amount received; \$7060 — \$6604 = \$456, gain.

Or, \$8 × 36 + \$10½ × 16 = \$456.

On 36 shares he received \$8 a share more than he gave, and on 16 shares \$10½ a share more than he gave.

15. \$100 × 52 = \$5200; \$5200 × .80 = \$4160; \$4160 ÷ 1.04 = \$4000, or 40 shares.

16. \$50 × 84 = \$4200; 1.00$\frac{3}{7}$ — .94 = .06$\frac{3}{7}$, per cent gain;
\$4200 × .06$\frac{3}{7}$ = \$270.

19. \$125000 × .04 = \$5000; \$5000 + \$3500 = \$8500.

20. \$9051998 ÷ \$106686116 = .0848 + = 8.48 + %.

Page 157. (Continued.)

22. $70680 — $40325 = $30355, net earnings;
$30355 — $6505 = $23850, amount divided;
$23850 ÷ $450000 = $.05\frac{3}{10} = 5\frac{3}{10}\%$.

25. $.95\frac{5}{8}\% + \frac{1}{8}\% = 95\frac{3}{4}\%$, cost of stock;
$6894 ÷ .9575 = $7200, or 72 shares.

Commission for buying or selling stocks is computed on their par value or face, and the rate of brokerage is *added to* the quoted price before dividing. This is an exception to the general principle. Comp. Arith., Art. 231.

Page 158.

26. $5\frac{1}{2}\% + 106\% = 111\frac{1}{2}\%$, of the par value, what he received;

$111\frac{1}{2}\% - 108\frac{1}{2}\% = 3\%$, gain; $8400 × .03 = $252.

27. 4% + 5% = 9%; $450 ÷ .09 = $5000.

Buying at 4% *below* par and selling at 5% *above* par, he gained 9% of the par value of the stock. Hence $450 was 9% of the par value of the stock, and the par value was $5000, or 50 shares.

28. $124\% - 120\% = 4\%$; $.04 \div 1.20 = .03\frac{1}{3} = 3\frac{1}{3}\%$.

Buying at 120 and selling at 124, he gained 4% of the face or par value; but 4% is only $.03\frac{1}{3}$, or $3\frac{1}{3}\%$ of 120%, the cost price; and hence he gained $3\frac{1}{3}\%$ on his investment.

Page 160.

3. $\frac{2}{3}$ of $6000 = $4000, amount insured; 1% of $4000 is $40, and $\frac{1}{4}\%$ of $4000 is $\frac{1}{4}$ of $40, or $10; $40 + $10 = $50, the premium paid.

Page 160. (Continued.)

7. If \$240 was 2% of the goods insured, 1% was \$120, and 100% was \$12000.

9. $\$7850 \times .00\frac{3}{4} = \$58.875$, premium on store;
$\$12400 \times .00\frac{1}{2} = \$62.$, premium on goods;
$\$58.875 + \$62. = \$120.875$, total premium.

Page 161.

13. $\$93.60 \div .00\frac{3}{5} = \$15600.$

15. $\$150000 \times .00\frac{3}{4} = \$1125$, premium on block;
$\$40000 \times .00\frac{7}{8} = \$350$; $\$35000 \times .00\frac{4}{5} = \$280$;
$\$350 + \$280 = \$630$, total paid for reinsuring;
$\$1125 - \$630 = \$495.$

16. $\$135000 \times \frac{4}{5} = \$108000$, whole risk;
$\$108000 \times \frac{1}{3} \times .00\frac{3}{4} = \$270$, prem. of 1st company;
$\$108000 \times \frac{2}{5} \times .00\frac{7}{8} = \$378$, " " 2d "
$\$108000 \times \frac{4}{15} \times .00\frac{2}{3} = \$192$, " " 3d "
$\$270 + \$378 + \$192 = \$840$, total premium.

17. $\frac{1}{3} + \frac{2}{5} = \frac{11}{15}$; $\frac{15}{15} - \frac{11}{15} = \frac{4}{15}$, 3d Co.'s part of risk;
$\$60000 \times \frac{1}{3} = \$20000$, loss of 1st company;
$\$60000 \times \frac{2}{5} = \$24000$, " " 2d "
$\$60000 \times \frac{4}{15} = \$16000$, " " 3d "

Each company loses the same part of the whole loss as it took part of the whole risk.

18. $\$3500 \times .00\frac{3}{5} \times 10 = \$210$, ten years' premiums;
$\$3500 - \$210 = \$3290.$

19. $\$40000 \times .02\frac{1}{2} \times 20 = \$20000$, premiums for twenty years;
$\$40000 - \$20000 = \$20000$, loss.

Page 162.

21. $21825 ÷ .97 = $22500.

[See suggestion under previous problem.]

25. $35000 × .99 = $34650, value of goods.

Or, since $35000 is the amount insured, 1% of it, or $350, is the premium paid, therefore $35000 — $350 = $34650, value of goods.

26. $50400 × $.03\frac{1}{2}$ = $1764, premium;
$50400 — $1764 = $48636, value of flour.

Page 163.

29. $44.50 × 5 = $222.50, yearly premium;
$222.50 × 10 = $2225, sum of premiums.

Page 164.

31. $21.70 × 8 = $173.60, annual premium;
$173.60 × 6 = $1041.60, total premiums paid;
$8000 — $1041.60 = $6958.40.

Page 165.

3. $1.50 × 540 = $810; $19340.20 — $810 = $18530.20, tax to be assessed on property;
$18530.20 ÷ $1425400 = .013 = $1\frac{3}{10}$%, or 13 mills.

5. $11703.845 ÷ .985 = $11882.076 +, tax collected;
$11882.076 + ÷ .925 = $12845.487 +, tax assessed.
Or, $1\frac{1}{2}$% + $7\frac{1}{2}$% = 9%; 1.00 — .09 = .91;
$11703.845 ÷ .91 = $12861.368.

[See note to problem 4, p. 165.]

Page 165. (Continued.)

6. $145850 × .90 = $131265, tax collected;
$131265 × .99¼ = $130280.51¼, net proceeds.

Or, $145850 × .90 = $131265, tax collected;
$131265 × .00¾ = $984.48¾, collection fee;
$131265 — $984.48¾ = $130280.51¼, net proceeds.

Page 166.

8. $35460850 × .02½ = $886521.25, tax assessed;
$886521.25 × .92 = $815599.55, tax collected;
$815599.55 × .97 = $791131.56 +, net proceeds.

11. $3570 — $1650 = $1920; $1920 × .05 = $96.

12. $9850 × .012¼ = $120.662, tax on property;
$2750 × .05 = $137.50, income tax;
$120.662 + $137.50 = $258.162, total tax.

Page 169.

3. 250 lb. × 340 = 85000 lb.; 85000 lb. × .95 = 80750 lb., net weight of coffee;
80750 × $.05 = $4037.50, duty.

5. 475 lb. × 225 = 106875 lb.; 106875 lb. × .87½ = 93515⅝ lb., net weight of sugar;
93515⅝ × $.03 = $2805.468, duty on sugar;
126 gal. × 120 = 15120 gal.; 15120 gal. × .98 = 14817.6 gal., net amount of molasses;
14817.6 × $.08 = $1185.408, duty on molasses;
$2805.468 + $1185.408 = $3990.876.

Page 169. (Continued.)

7. $\$2.15 \times 1450 = \$3117.50$, invoice price of broadcloth;
$\$1.60 \times 3240 = \$5184.00$, " " " carpet;
$\$3117.50 + \$5184 = \$8301.50$, " " " wool'n gds.;
$\$8301.50 \times .35 = \$2905.525$, duty on woolen goods;
$\$2.85 \times 480 = \$1368$, invoice price of silks;
$\$1368 \times .50 = \$684$, duty on silks;
$\$2905.525 + \$684 = \$3589.525$, total duty.

8. $\$1100 \div .40 = \$2750$; $\$2750 \div 1250 = \$2.20$, invoice price a yard.
$\$2.20 \times 1.40 = \$3.08$; $\$3.08 \times 1.20 = \$3.696$.

As the duty was 40%, the whole cost per yard was 40% more than \$2.20, which is \$3.08; in order to gain 20% he must sell at 20% higher than \$3.08, or at \$3.696.

Page 170.

3. $\$3500 + \$1250 + \$3750 + \$1000 + \$2500 = \$12000$, liabilities;
$\$7150 - \$550 = \$6600$, net proceeds of assets;
$\$6600 \div \$12000 = .55$, or 55%.
$\$3500 \times .55 = \$1925$, A's;
$\$1250 \times .55 = \$687.50$, B's;
$\$3750 \times .55 = \$2062.50$, C's;
$\$1000 \times .55 = \$550$, D's;
$\$2500 \times .55 = \$1375$, E's.

Page 171.

4. $\$9500 + \$5400 + \$2100 = \$17000$;
$\$17000 \times .95 = \$16150$, net proceeds,
$\$16150 \div \$25000 = .646$, or $64\frac{3}{5}$ cents on a dollar.

INTEREST.

Page 173.

16. The interest of \$50 for 1 year, at 6%, is \$3, and the interest for 4 years 2 months, or $4\frac{1}{6}$ years, is $4\frac{1}{6}$ times \$3, which is \$12.50; the interest for 6 years 10 months, or $6\frac{5}{6}$ years, is $6\frac{5}{6}$ times \$3, which is \$20.50.

18. The interest of \$500 for 1 year, at 5%, is \$25, and for 3 years 2 months, or $3\frac{1}{6}$ years, the interest is $3\frac{1}{6}$ times \$25, which is $\$79\frac{1}{6}$.

23. (1) $\$1000 \times .03\frac{1}{2} = \$35$; $\$35 \times 1\frac{1}{12} = \$37.91\frac{2}{3}$;
(2) $\$1000 \times .073 = \$73$; $\$73 \times 1\frac{1}{12} = \$79.08\frac{1}{3}$.

24. (1) $\$85 \times .07 = \$5.95$; $\$5.95 \times 3\frac{7}{12} = \$21.32\frac{1}{12}$;
(2) $\$5.95 \times \frac{3}{4} = \$4.462 +$.

Page 174.

30. 4 yr. 3 mo. 27 da. = 4 yr. 3.9 mo. = 4.325 yr.;
$\$321.70 \times .12\frac{1}{2} = \$40.2125$, interest for 1 year;
$\$40.2125 \times 4.325 = \$173.919$, int. for 4 yr. 3 mo. 27 da.

35. 1 yr. 6 mo. 12 da. = 1 yr. 6.4 mo. = $1.5\frac{1}{3}$ yr.;
$\$80.65 \times .10\frac{1}{2} = \$8.46825$, interest for 1 year;
$\$8.46825 \times 1.5\frac{1}{3} = \$12.984$, int. for 1 yr. 6 mo. 12 da.;
$\$80.65 + \$12.984 = \$93.634$, amt. for 1 yr. 6 mo. 12 da.

39. \$1262.70
$.07\frac{1}{4}$
120) \$91.54575, interest for 1 year;
.76288, " " 3 days;
\$90.78287, " " 11 months 27 days.

The interest for 11 months 27 days is found by subtracting the interest for 3 days ($\frac{3}{360}$ mo. = $\frac{1}{120}$ yr.) from the interest for 1 year.

Page 174. (Continued.)

43. $2016.05
.08
6) $161.2840, interest for 1 year;
60) 26.880, " " 2 months;
$188.164, " " 1 year 2 months;
.448, " " 1 day;
$187.716, " " 1 year 1 month 29 days.

The interest for 1 year 1 month 29 days is found by subtracting the interest for 1 day from the interest for 1 year 2 months. The interest for 1 day is $\frac{1}{60}$ of the interest for 2 months.

44. $192.60
.05
$9.6300, interest for 1 year;
$9.63 × 2 = $19.26, interest for 2 years;
$9.63 × $\frac{1}{6}$ = 1.605, " " 2 months;
$1.605 × $\frac{1}{30}$ = .0535, " " 2 days;
$20.9185, " " 2 yr. 2 mo. 2 da.

Page 175.

47. 1869 11 28
1864 6 3
5 yr. 5 mo. 25 da.
$565.80
.08 $565.80, principal.
12) $45.2640 × 5 = $226.32, interest for 5 years;
3.772 × 5 = 18.86, " " 5 months;
15 da. = $\frac{1}{2}$ mo. 1.886, " " 15 days;
10 da. = $\frac{1}{3}$ mo. 1.257, " " 10 days.
$814.123, amount of $565.80 from June 3, 1864 to Nov. 28, 1869, at 8%.

Page 178.

32. Interest of $1 for 2 years 15 days, at 6% = $.1225;

$142.64
.1225
71320
28528
28528
14264
4) $17.4734, interest at 6%.
4.3683
$13.1051, interest at 4½% (¼ less than 6%).

The interest at 4½% may also be found by taking ¾ of the interest at 6%.

42.

1870	5	6
1866	9	12
3 yr.	7 mo.	24 da.

Interest of $1 at 6% = $.219.

$108.60 × .219 = $23.7834, interest at 6%.
⅓ of $23.7834 = 7.9278
$31.7112, interest at 8%.

44.

1868	4	11
1865	10	17
2 yr.	5 mo.	24 da.

Interest of $1 at 6% = $.149.

$350 × .149 = $52.15, interest at 6%.
⅙ of $52.15 = 8.69
$ 60.84, interest at 7%.
350.
$410.84, amount.

46.

1870	6	1
1868	7	5
1 yr.	10 mo.	26 da.

Page 178. (Continued.)

Interest of $1 at 6% = $.114$\frac{1}{3}$.

$1250 × .114$\frac{1}{3}$ = $142.9166, interest at 6%.
$\frac{1}{3}$ of $142.9166 = 47.6388
$ 190.555, interest at 8%.
1250
$1440.555, amount.

48. 1870 6 25
1868 2 20
2 yr. 4 mo. 5 da.

Interest of $1 at 6% = $.140$\frac{5}{6}$.

$65.80 × .140$\frac{5}{6}$ = $9.2668, interest at 6%.
$\frac{1}{6}$ of $9.2668 = 1.5444
$10.811, interest at 7%.
65.80
$76.611, amount.

Page 180.

4. In Dec. 3 days.
" Jan. 31 "
" Feb. 29 " (leap year).
" Mar. 15 "
6) 78 "
13 mills.

$ 65.80
.013
19740
6580
6) $.85540, int. at 6%.
.14256, " " 1%.
.7128, int. at 5%.
$65.80
$66.5128, amt. at 5%.
$.14256 × 10 = $1.4256, int. at 10%.
65.80
$67.2256, amt. at 10%.

Page 180. (Continued.)

9. In Jan. 16 days.
“ Feb. 29 “ (leap year).
“ Mar. 31 “
“ Apr. 30 “
“ May 31 “
“ June 21 “
6)158 “
26⅙, mills.

$80
.026⅓
3)$2.1066, int. at 6%.
.7022
$2.8088, int. at 8%.
80
$82.8088, amount.

11. In May 19 days.
“ June 30 “
“ July 24 “
6)73 “
12⅙, mills.

$500
.012⅙
6)$6.0833, int. at 6%.
1.0138
$7.097, int. at 7%.
500
$507.097, amount.

Page 183.

2. 1867 12 4
1867 6 10
$50. 5 mo. 24 da. .029.

1868 3 25
1867 12 4
3 mo. 21 da. .018½.
$12.

1868 7 9
1868 3 25
3 mo. 14 da. .017⅓.
$75.

$600
.029
$17.40, 1st interest.
600.
$617.40
50. 1st payment.
567.40
.018½
28370
453920
56740
$10.4969, 2d interest.
567.40
$577.8969

Page 183. (Continued.)

$577.8969
1868 10 15 — 12. 2d payment.
1868 7 9 — $565.8969
3 mo. 6 da. .016. — $.017\frac{1}{3}$
1886323
39612783
5658969
$9.8088796, 3d interest.
565.8969
$575.7058
75. 3d payment.
$500.7058
.016
30042348
5007058
$8.0112928, 4th interest.
500.7058
$508.7171, due Oct. 15, 1868.

3. 1865 11 10 — $1000
1864 4 10 — .095
1 yr. 7 mo. .095. — 6)95.000
$80.50. — 15.8333
$110.8333, 1st interest.
1866 7 5 — $1000
1865 11 10 — $.039\frac{1}{6}$
7 mo. 25 da. $.039\frac{1}{6}$. — 6)39.1667
$100. — 6.5278
$45.6945, 2d interest.
1867 1 10 — 110.8333, 1st interest.
1866 7 5 — 1000.
6 5 da. $.030\frac{5}{6}$. — $1156.5278
$450.80.

(Concluded on p. 44.)

Page 183. (Continued.)

$1156.5278
1869 10 1 180.50, 1st & 2d payt.
1867 1 10 $976.0278
2 yr. 8 mo. 21 da. .163½. .030⅚
8133565
$500. 29280834
6)$30.0941905
5.0157
1870 1 1 $35.1098, 3d interest.
1869 10 1 976.0278
3 mo. .015. $1011.1376
450.80, 3d payment.
$560.3376
.163½
2801688
16810128
33620256
5603376
6)$91.6151976
15.26919
$106.8844, 4th interest.
560.3376
$667.222
500. 4th payment.
167.222
.015
836110
167222
6)$2.50833
.41805
$2.9264, 5th interest.
167.222
$170.148, amount due Jan. 1, 1870.

Page 183. (Continued.)

4.

	1869	1	20	
	1868	7	4	
$15.		6 mo.	16 da.	$.032\frac{2}{3}$.
	1869	6	9	
	1869	1	20	
$200.		4 mo.	19 da.	$.023\frac{1}{6}$.
	1869	10	20	
	1869	6	9	
$10.		4 mo.	11 da.	$.021\frac{5}{6}$.
	1870	1	10	
	1869	10	20	
		2 mo.	20 da.	$.013\frac{1}{3}$.

$450.
$.032\frac{2}{3}$
300
900
1350
6)$14.700
2.45
10
$24.50, 1st interest.
$450.
$.023\frac{1}{6}$
75
1350
900
6)$10.425
1.7375
10
$17.375, 2d interest.
24.50, 1st interest.
450.
$491.875

(Concluded on p. 46.)

Page 183. (Continued.)

$491.875
215. 1st and 2d [payt.
$276.875
$.021\frac{5}{6}$
6)6.045104
1.007517
10
$10.0751, 3d int.
$276.875

$276.875
$.013\frac{1}{3}$
6)3.691667
.615278
10
$6.15278, 4th int.
10.0751, 3d int.
276.875
$293.10288
10. 3d payt.
$283.103, amount due.

5. 1865 9 1
1865 3 4
$12. 5 mo. 27 da. $.029\frac{1}{2}$.

1866 5 4
1865 9 1
$10. 8 mo. 3 da. $.040\frac{1}{2}$.

1866 9 15
1866 5 4
$250. 4 mo. 11 da. $.021\frac{5}{6}$.

1867 1 20
1866 9 15
$400. 4 mo. 5 da. $.020\frac{5}{6}$.

1868 7 1
1867 1 20
1 yr. 5 mo. 11 da. $.086\frac{5}{6}$.

$850
$.091\frac{5}{6}$
$78.0583, 1st, 2d, 3d, [int.
850.
$928.0583
272. 1st, 2d, 3d, [payt.
$656.0583
$.020\frac{5}{6}$
$13.6678812, 4th int.
656.0583
$669.7262
400. 4th payt.
$269.7262
$.086\frac{5}{6}$
$23.4212250, 5th int.
269.7262
$293.147, amount due.

The 1st multiplier, $.091\frac{5}{6}, = .029\frac{1}{2} + 040\frac{1}{2} + .021\frac{5}{6}$. See note 1, page 183, Complete Arithmetic.

Page 184.

6. 1867	12	6		$520.	
1867	4	12		.039	
$120.	7 mo.	24 da.	.039.	2)20.28	
				10.14	
1868	7	9		$30.42,	1st interest.
1867	12	6		520.	
$12.	7 mo.	3 da.	.035½.	$550.42	
				120.	1st payment.
1868	11	30		$430.42	
1868	7	9		.084⅙	
$9.	4 mo.	21 da.	.023½.	2)36.22702	
				18.11351	
1869	5	1		$54.3405,	2d, 3d, 4th,
1868	11	30		430.42	[int.
	5 mo.	1 da.	.025⅙.	$484.7605	
				21.	2d and 3d payt.
				$463.761,	amount due.

7. 1868	9	14		$1250.	
1868	7	1		.012⅙	
	2 mo.	13 da.	.012⅙.	4)15.208	
$300.				3.802	
				$19.010,	1st interest.
1869	1	20		1250.	
1868	9	14		$1269.01	
	4 mo.	6 da.	.021.	300.	1st payment.
$12.				$969.01	
				.069	
1869	10	20		4)66.86169	
1869	1	20		16.71542	
	9 mo.	0 da.	.045.	$88.577,	2d, 3d, 4th,
$20.				969.01	[int.
				$1052.587	

(Concluded on p. 48.)

Page 184. (Continued.)

1869	11	8		$1052.587	
1869	10	20		532.	2d, 3d, 4th,
		18 da.	.003.	$520.587	[payt.
$500.				.008⅚	
				4)4.598518	
1870	1	1		1.149629	
1869	11	8		$5.748,	5th interest.
	1 mo.	23 da.	.008⅚.	520.587	
				$526.335,	amount due.

8. 1867	7	28		$1000.	
1867	4	10		.018	
	3 mo.	18 da.	.018.	6)18.000	
$500.				3.00,	int. at 1%.
				$30.00,	1st interest.
1867	12	13		1000.	
1867	7	28		$1030.00	
	4 mo.	15 da.	.022½.	500.	1st payment.
$8.				$530	
				.056½	
1868	2	25		6)29.945	
1867	12	13		4.9908	
	2 mo.	12 da.	.012.	$49.908	2d, 3d, 4th,
$12.				530.	[int.
				$579.908	
1868	7	7		145.00,	2d, 3d, 4th,
1868	2	25		$434.908	[payt.
	4 mo.	12 da.	.022.	.014⅓	
$125.				6)6.233681	
				1.038947	
				$10.389,	5th interest.

Page 184. (Continued.)

1868 10 3	$10.389,	5th interest.
1868 7 7	434.908	
2 mo. 26 da. .014⅓.	$445.297	
$200.	200.	5th payment.
	$245.297	
1869 3 15	.027	
1868 10 3	6)6.623019	
5 mo. 12 da. .027.	1.103837	
$50.	$11.038,	6th interest.
	245.297	
1869 6 3	$256.335	
1869 3 15	50.	6th payment.
2 mo. 18 da. .013.	$206.335	
	.013	
	6)2.682355	
	.447059	
	$4.471,	7th interest.
	206.335	
	$210.806,	amount due.

Page 185.

9. Principal,		$800.
Interest for 9 months 18 days, at 8%,		51.20
		$851.20
First payment,	$200.	
Interest for 7 months 15 days,	10.	
Second payment,	75.	
Interest for 4 months 20 days,	2.333	
Third payment,	125.	
Interest for 2 months 10 days,	1.944	
		414.277
Amount due December 30, 1869,		$436.923

Page 185. (Continued.)

10. Principal,		$350.
Interest for 9 months, at 6%,		15.75
		$365.75
First payment,	$45.	
Interest for 7 months 11 days,	1.657	
Second payment,	60.	
Interest for 6 months,	1.80	
Third payment,	80.	
Interest for 3 months 26 days,	1.547	
Fourth payment,	50.	
Interest for 1 month,	.25	
		240.254
Amount due November 1, 1868,		$125.496

Page 186.

6. In May 16 days.	$80.25	
“ June 30 “	.022	
“ July 31 “	16050	
“ Aug. 31 “	16050	
“ Sept. 24 “	3)$1.76550,	int. at 6%.
6)132 “	58850	
22, mills.	$2.354,	int. at 8%.

See note on page 185 of Complete Arithmetic.

8. In Apr. 10 days.	$7.50	
“ May 31 “	$.029\frac{1}{6}$	
“ June 30 “	3)$.21875,	int. at 6%.
“ July 31 “	.0729	
“ Aug. 31 “	$.291,	int. at 8%.
“ Sept. 30 “	7.50	
“ Oct. 12 “	$7.791,	amount.
6)175 “		
$29\frac{1}{6}$, mills.		

Page 187.

14. 1869	4	27		$240		
1868	2	15		.072		
1 yr.	2 mo.	12 da.	.072	6)$17.28,	int. at	6%.
				$2.88,	" "	1%.

$23.04 ÷ $2.88 = 8, the rate.

16. $407.088 — $345.60 = $61.488, interest;
$345.60 × .152½ = $52.704, int. at 6%, for 2 yr. 6 mo. 15 da.;
$52.704 ÷ 6 = $8.784, interest at 1%;
$61.488 ÷ $8.784 = 7, rate.

The difference in dates is 2 years 6 months 15 days.

20. $300 × .06 = $18, interest for 1 year;
$347.25 — $300 = $47.25, interest;
$47.25 ÷ $18 = 2.625, years;
2.625 yr. = 2 yr. 7.5 mo. = 2 yr. 7 mo. 15 da.

Page 188.

23. $1 ÷ $.04 = 25, yr.; $1 ÷ $.06 = 16⅔, yr.; $1 ÷ $.10 = 10, yr.

This is the same as, In what time will $1 amount to $2, at 4% interest? or, In what time will the interest on $1, at 4%, be $1? $1, the interest, divided by $.04, the interest on the principal, $1, for 1 year, gives 25. It will take a sum 25 years to double itself, at 4% interest, 16⅔ years, at 6%, and 10 years, at 10%.

NOTE.—It is evident that any other sum than $1 might be used for illustration.

Page 188. (Continued.)

25. (1) $2 ÷ .05 = 40; 40 years.
(2) $2 ÷ .10 = 20; 20 years.
(3) $2 ÷ .06 = 33⅓; 33⅓ years.

This is the same as, In what time will the interest on $1, at 5%, be $2?

Page 189.

32. Int. on $1, for 4 mo. 12 da., at 8% = $.029⅓;
$220. ÷ .029⅓ = $7500.

33 In Jan. 21 days.
" Feb. 29 " (leap yr.).
" Mar. 13 "
6)63 "
10½, mills.

3)$.0105, int. of $1, at 6%.
.0035, " " " " 2%.
$.007, " " " " 4%.
$17.78 ÷ .007 = $2540.

Page 190.

36. 6)$.043, interest of $1, at 6%.
7⅙
$.050⅙, interest of $1, at 7%.
$1.050⅙, amount of $1, at 7%.

$567.09 ÷ $1.050⅙ = 540; $1 × 540 = $540.

37. $.17, interest of $1, at 8%.
$1.17, amount of $1, at 8%.

$421.20 ÷ $1.17 = 360; $1 × 360 = $360.

38. Amt. of $1, for 1 yr. 4 mo. 21 da., at 6% = $1.0835;
$2600.40 ÷ $1.0835 = 2400; $1 × 2400 = $2400.

Page 191.

3. Int. of \$1, for 8 mo. 18 da., at 6% = \$.043.
\$512.60
.043
6)22.04180, interest at 6%.
$\$3.67363\frac{1}{3}$, “ “ 1%.
$\$25.72 \div \$3.6736\frac{1}{3} = 7 +$; $= 7 + \%$.

6. \$326.50
.08
\$26.1200, interest for 1 year.
\$2.76 ÷ \$26.12 = .10567; .10567 yr. = 1 mo. 8 + da.

10. 6)\$.04, interest of \$1, for 8 months, at 6%.
$.00\frac{2}{3}$
$\$.04\frac{2}{3}$, “ “ “ “ “ “ “ 7%.
$\$1.04\frac{2}{3}$, amount “ “ “ “ “ “ “
$\$659.40 \div \$1.04\frac{2}{3} = 630$; $\$1 \times 630 = \$630$.

14. Interest on \$1, for 292 days, at 7% = $\$.056\frac{7}{9}$;
$\$21.757 \div .056\frac{7}{9} = \$383.195$.

17. Interest on \$1, for 1 year, at 7% = \$.07;
\$1000 ÷ .07 = \$14285.714.

Or, \$1000 ÷ \$.07 = 14285.714;
\$1 × 14285.714 = \$14285.714.

Page 193.

10. \$216 ÷ 1.16 = \$186.206+; \$216 — \$186.206 = \$29.794, true discount; \$216 × .16 = \$34.56, interest; \$34.56 — \$29.794 = \$4.766.

NOTE.—In this and the following six problems, the true discount is to be computed on the given principal due in

Page 193. (Continued.)

the given time *without interest*—a fact implied, but not definitely stated in the earlier editions of the Complete Arithmetic.

17. $442 ÷ $1.105 = 400; $1 × 400 = $400.

The problem is equivalent to this, "What is the present worth of $442, due 1 year 6 months hence, without interest, current rate 7%?"

It may also be considered as a problem under "Problem V," the debt being the *amount* and the face of the note the principal.

8. $371 ÷ $1.06 = 350; $1 × 350 = $350;
$350 — $345 = $5.

Page 196.

13. $1319.50 ÷ 1.015 = $1300;
$1319.50 — $1300 = $19.50, true discount;
$1319.50 × .015 = $19.7925, bank discount;
$19.7925 — $19.50 = $.29+.

NOTE.—In a State where grace is allowed, a note *payable* after 60 days is not legally *due* until the expiration of 63 days. It may be paid, if the maker so chooses, in 60, 61, 62, or 63 days, but it must be paid, or go to protest, in 63 days. See note on page 194 of Complete Arithmetic. Hence in problems 13, 14, and 15, the given time *includes the 3 days of grace.* The true discount is to be computed on the given principal due in the given time, *without interest.* See note under the solution of problem 10, page 53.

15. $3330.80 ÷ 1.00875 = $3301.908, present worth;
$3330.80 — $3301.908 = $28.892, true discount;

Page 196. (Continued.)

$\$3330.80 \times .008\frac{3}{4} = \$29.1445$, bank discount;
$\$29.1445 - \$28.892 = \$.25+$, difference.

20. $\$125 \times 1.0105 = \$126.312$, amt. of $125 for 63 days;
$\$126.312 \times .016\frac{7}{18} = \$2.070$, bank discount for 59 days, at 10%.
$\$126.312 - \$2.070 = \$124.242$, proceeds.

NOTE. — The discount is computed on the *amount* of the note at the time of its *maturity*, and for the time between the date of discounting and the date of maturity.

22. $\$90 \times 1.017\frac{1}{12} = \$91.5375$, amount for 4 months 3 days, at 5%.
$\$91.5375 \times .014\frac{7}{12} = \$1.3349$, bank discount for 75 days, at 7%;
$\$91.5375 - \$1.3349 = \$90.20+$, proceeds.

Page 197.

23. Principal,		$650.
Interest for 1 year 1 month 2 days, at 6%,		42.466
		$692.466
First payment,	$300.	
Interest for 6 months 3 days,	9.15	
		309.15
		$383.316

$\$383.316 \times .01 = \$3.833$, bank dis. for 60 days, at 6%;
$\$383.316 - \$3.833 = \$379.483$, proceeds.

26. $\$1 - \$.030\frac{3}{4} = \$.969\frac{1}{4}$; $\$811 \div \$.969\frac{1}{4} = 836.729$;
$\$1 \times 836.729 = \$836.729$.

27. $\$1 - \$.031 = \$.969$; $\$1938 \div \$.969 = 2000$; $\$1 \times 2000 = \$2000$.

Page 197. (Continued.)

28. $\$750 \times .03\frac{1}{3} = \$25$, discount; $\$750 - \$25 = \$725$.

29. $\$340 \times .06 = \$20.40$; $\$340 - \$20.40 = \$319.60$.

30. $\$1030 \times .05 = \$51.50$; $\$1030 - \$51.50 = \$978.50$.

31. $\$950 \times .07\frac{1}{2} = \$71.25$; $\$950 - \$71.25 = \$878.75$.

32. $\$3540 \times .05 = \$177$, discount for cash;
$\$3540 - \$177 = \$3363$, cash value;
$\$3363 \div .97\frac{4}{9} = \$3451.197$, face of note;
$\$3540 - \$3451.197 = \$88.803$, gain.

The number of days in 3 months from March 20th is 92, and to obtain $3363 from bank, he must give his note for a sum which, discounted at 10% for 92 days (including grace), will give $3363 as proceeds. The note would be drawn payable in 3 months without grace.

Page 203.

6. $\$650 \times .014 = \$9.10$; $\$650 - \$9.10 = \$640.90$, proceeds;
$\$650 \times .00\frac{1}{2} = \$3.25$, premium; $\$640.90 + \$3.25 = \$644.15$.

7. $\$1 - \$.009\frac{1}{3} - .00\frac{3}{4} = \$.983\frac{1}{6}$, cost of $1;
$\$320 \times .983\frac{1}{6} = \$314.613$.

10. $\$1256.25 \div 1.005 = \$1250$.

12. $\$1 - \$.014 = \$.986$, proceeds of $1;
$\$.986 + \$.0125 = \$.9985$, cost of $1;
$\$798.80 \div .9985 = \$800$, face of draft.

13. $\$1 - \$.0055 = \$.9945$, proceeds of $1;
$\$.9945 - \$.015 = \$.9795$, cost of $1;
$\$352.62 \div .9795 = \$360$, face of draft.

Page 204.

1. $\$500 \times 3 = \$1500$; $\$1500 \times 1.09\frac{1}{2} = \$1642.50$.

3. \$1.05, cost of \$1 in bonds; $\$4725 \div 1.05 = \$4500$.

Page 205.

5. Since \$1 in gold will buy \$1.15 in currency, \$8500 in gold will buy 8500 times \$1.15, or \$9775.

6. $\$1400 \div \$1.12 = 1250$; $\$1 \times 1250 = \$1250$.

Since \$1.12 in currency will buy \$1 in gold, as many dollars in gold can be bought for \$1400 as \$1.12 is contained times in \$1400.

8. $\$1000 \times 1.15 = \$1150$, currency value of bond;
$\$1150 \div 1.20 = \$958.33\frac{1}{3}$, gold value of bond;

10. $\$1.00 \div 1.125 = \$.88\frac{8}{9}$, value in gold.

Since $\$1.12\frac{1}{2}$ in currency will buy \$1 in gold, as many times \$1 in gold can be bought for \$1 in currency as $\$1.12\frac{1}{2}$ is contained times in \$1, which is .888+.

11. $\$9500 \times .025 = \$237.50$, interest in gold;
$\$237.50 \times 1.15 = \$272.125$, value of interest in currency.

12. $5\% = .05$; $.05 \times 1.20 = .06 = 6\%$.

13. $\$100 \times .03 = \$3$, int. in gold on a \$100 bond every six months;
$\$3 \times 1.20 = \$3.60$, currency value of gold interest;
$\$3.60 \div 106 = .03\frac{21}{53} = 3\frac{21}{53}\%$, semi-annual rate;
$3\frac{21}{53}\% \times 2 = 6\frac{42}{53}\%$, annual rate.

Page 205. (Continued.)

The interest on a \$100 U. S. 5-20 bond is \$3 in gold every six months, and, when gold is at 120, \$3 in gold is worth \$3.60 in currency. But each \$100 bond cost the college \$106, and hence when gold is at 120, it receives \$3.60 semi-annually on each \$106 of its investment, which is $3\frac{21}{53}\%$. When gold is at 110, it receives \$3.30 in currency semi-annually on \$106, which is $3\frac{6}{53}\%$. When gold is at par, it receives only \$3 in currency semi-annually on \$106, which is $2\frac{44}{53}\%$. The annual rate of interest when gold is 120, 110, and at par is respectively, $6\frac{42}{53}\%$ ($3\frac{21}{53}\% \times 2$), $6\frac{12}{53}\%$, and $5\frac{35}{53}\%$.

Note.—The last step in the above solution is based on the supposition that 6% interest, payable semi-annually, is the same as 6% interest, payable annually, which is not correct. The interest each year of \$100, at 6%, payable semi-annually, is $\$3 + \$3 + \$3 \times .03 = \$6.09$, and hence 6% interest, payable semi-annually, equals $6\frac{9}{100}\%$, payable annually. But in commercial circles the annual rate is supposed to be *twice* the semi-annual rate.

Page 207.

4. $\$3500 \times .06 = \$210$, int. on principal, due annually.
$\$210 \times 4 = \$840$, total annual interest.
$\$210 \times .06 \times 3 = 37.80$, simp. int. on 1st an. int. for 3 yrs.
$\$210 \times .06 \times 2 = 25.20$, " " " 2d " " " 2 "
$\$210 \times .06 \times 1 = 12.60$, " " " 3d " " " 1 yr.
\$915.60, total interest due at maturity.
\$3500
\$4415.60, amount due at maturity.

5. $\$650 \times .07 = \$45.50$, int. on principal, due annually.
$\$45.50 \times 2\frac{1}{2} = \$113.75$, total annual interest.

Page 207. (Continued.)

45.50 × 2½ = $113.75, total annual interest.
45.50 × .07 × 1½ = 4.7775, int. on 1st an. int. for 1½ yrs.
45.50 × .07 × ½ = 1.5925, " " 2d " " " ½ yr.
$120.12, total int. due at maturity.
$650
$770.12, amount due at maturity.

8. $500 × .10 = $50, first annual interest.
$50 × 2 = $100, annual interest due Jan. 15, 1867.
$50 × .10 = 5, interest on 1st annual interest.
$500
$605, amount due Jan. 15, 1867.

$$\left.\begin{array}{ll}\text{Payt.} & \$100.00\\ \text{Int.} & 6.50\end{array}\right\} = \$106.50,$$ amt. of $100 Jan. 15, 1867.
$498.50, new principal.
$498.50 × .05 = 24.925, int. due July 15, 1867.
$523.425, amt. " " " "

$$\left.\begin{array}{ll}\text{Payt.} & \$200\\ \text{Int.} & 7\end{array}\right\} = \$207.$$ amt. of $200 July 15, 1867.
$316.425, amount due at maturity.

Page 209.

5. $650.
1.06
$689.00, amount due at the end of 1 year.
1.06
$730.34, amount due at the end of 2 years.
1.06
$774.1604, amount due at the end of the 3d year.
1.022
$791.1919288, amt. due at end of 3 yr. 4 mo. 12 da.
650
$141.191, comp. int. at end of 3 yr. 4 mo. 12 da.

Page 211.

7. $\$1.601032 \times 500 = \$800.516$.

The amount of any sum for 6 years, at 8%, compounded twice a year, is the same as the amount for 12 years, at 4%, compounded annually. The amount of \$1, at 4%, for 12 years, is \$1.601032, as found in table, p. 210.

8. \$1.628895, amt. of \$1 for 10 years. See table, p. 210.
1250
\$2036.11875, amt. of \$1250 at end of 10 years.
$.018\frac{3}{4}$
\$38.17722656, int. for 4 mo. 15 da.
2036.11875
\$2074.296, amt. at end of 10 yr. 4mo. 15 da.

Page 212.

4. $\$800 \times \frac{1}{8} = \$100;\ \$100 \times 2 = \$200.$
$\$800 \times \frac{1}{2} = \$400;\ \$400 \times 3 = \$1200.$
$\$800 \times \frac{3}{8} = \$300;\ \$300 \times 6 = \$1800.$
\$800)\$3200(4

The equated time is 4 months after date.

6. $\$500 \times 4 = \$2000$
$700 \times 6 = 4200$
$1000 \times 5 = 5000$
\$2200)\$11200($5\frac{1}{11}$

The equated time is $5\frac{1}{11}$ months after purchase.

7. \$500
$400 \times 3 = \$1200$
$500 \times 4 = 2000$
$600 \times 8 = 4800$
\$2000)\$8000(4

The equated time is 4 months after purchase.

Page 213.

11. $\$375$

$$\begin{array}{llll}
 & \$425 \times 30 & = & \$12750\\
 & \$500 \times 60 & = & \$30000\\
 & \underline{\$600} \times 75 & = & \underline{\$45000}\\
\$1900 & &) & \$87750(46.1 +
\end{array}$$

The equated time is 46 days after date.

Or, by interest:

$$\begin{array}{llll}
\$375 & & & \\
\$425 \times .005 & = \$2.125, & \text{interest for} & 30 \text{ days.}\\
\$500 \times .01 & = \$5.00, & \text{“ \quad “} & 60 \text{ “}\\
\underline{\$600} \times .0125 & = \underline{\$7.50}, & \text{“ \quad “} & 75 \text{ “}\\
\$1900 & \ \ \$14.625 & & \\
\underline{.000\frac{1}{6}} & & &
\end{array}$$

$\$.316\frac{2}{3}$, interest of $1900 for 1 day.

$\$14.625 \div \$.316\frac{2}{3} = 46 +.$

12. $\$340$

$$\begin{array}{llll}
 & 450 \times 31 & = & \$13950\\
 & 560 \times 66 & = & 36960\\
 & \underline{650} \times 92 & = & \underline{59800}\\
\$2000 & &) & \$110710(55.3
\end{array}$$

The equated time is 55 days from May 10, 1870, which is July 4, 1870.

13. $\$1000$

$$\begin{array}{llll}
 & 850 \times 30 & = & \$25500\\
 & 750 \times 92 & = & 69000\\
 & \underline{900} \times 122 & = & \underline{109800}\\
\$3500 & &) & \$204300(58.3
\end{array}$$

The equated time is 58 days forward from June 1, 1870, which is July 29, 1870.

Page 213. (Continued.)

14.

$$\begin{array}{rrl}
\$\ 75 & & \\
115 \times 20 & = & \$\ 2300 \\
220 \times 50 & = & 11000 \\
315 \times 71 & = & 22365 \\
\underline{350} \times 85 & = & \underline{29750} \\
\overline{\$1075} & &)\overline{\$65415}(60.8
\end{array}$$

The equated time is 61 days forward from May 6, 1870, which is July 6, 1870.

Page 215.

18.

$$\begin{array}{l}
\$\ 600 \times \quad 6 = \$3600; \\
\$\ 600 \times \quad 3 = \$1800; \\
\$1800 \div \quad 9 = \$\ 200; \\
\$\ 600 - \$200 = \$\ 400.
\end{array}$$

A is entitled to the use of $600 for 6 months, or $3600 for 1 month, and he has the use of $600 for 3 months or $1800 for 1 month, and is entitled to the use of $1800 more for 1 month, or $\frac{1}{9}$ of $1800, which is $200, for 9 months. Hence, at the end of 3 months, A must pay $600 — $200, or $400, if he wishes to extend the time for the payment of the balance 6 months, or to pay it 9 months after the date of the first payment.

Or, A loses the use of what he pays at the end of 3 months for 3 months, and thus gains the use of the balance for 6 months, or twice as long; hence the deferred payment should be one-half the payment made at the end of 3 months. Thus the two payments together are three times the smaller, and their sum being $600, the smaller payment is $\frac{1}{3}$ of $600, or $200; $600 — $200 = $400, payment at the end of 3 months.

Page 215. (Continued.)

19.

$$\begin{array}{rrr} & \$400 \times 8 = & \$3200 \\ & \underline{500 \times 6} = & \underline{3000} \\ \$1500 - & \overline{\$900} = \$600 &)\overline{\$6200}(10\tfrac{1}{3} \end{array}$$

The balance should be paid $10\frac{1}{3}$ months after maturity.

20.

$$\begin{array}{rrcl} & \$650 \times & 94 & = \$61100 \\ & 500 \times & 71 & = 35500 \\ & \underline{350 \times} & \underline{51} & = \underline{17850} \\ \$2500 - & \overline{\$1500} = \$1000 & &)\overline{\$114450}(114.45 \end{array}$$

The balance should be paid 114 days forward from July 10, 1870, the day of maturity of the bill, which gives November 1, 1870.

Page 216.

$$\begin{array}{llllrcl} \textbf{2.}\ \text{Due} & \text{July} & 15, & 1869, & \$400 & & \\ \text{“} & \text{Sept.} & 20, & \text{“} & 245 \times & 67 = & \$16415 \\ \text{“} & \text{Oct.} & 25, & \text{“} & 375 \times & 102 = & 38250 \\ \text{“} & \text{Dec.} & 15, & \text{“} & \underline{625} \times & 153 = & \underline{95625} \\ & & & & \overline{\$1645} & &)\overline{\$150290}(91.3 \end{array}$$

The equated time of maturity is 91 days forward from July 15, 1869, which is October 14, 1869.

$$\begin{array}{llllrcl} \textbf{3.}\ \text{Due} & \text{Sept.} & 9, & 1870, & \$340 & & \\ \text{“} & \text{Oct.} & 6, & \text{“} & 530 \times & 27 = & \$14310 \\ \text{“} & \text{Oct.} & 8, & \text{“} & 213 \times & 29 = & 6177 \\ \text{“} & \text{Dec.} & 30, & \text{“} & \underline{150} \times & 112 = & \underline{16800} \\ & & & & \overline{\$1233} & &)\overline{\$37287}(30.2 \end{array}$$

The equated time is 30 days forward from September 9, 1870, which is October 9, 1870.

Page 216. (Continued.)

4. Due June 3, 1868, $250
" Aug. 15, " 180 × 73 = $13140
" Sept. 20, " 325 × 109 = 35425
" Nov. 10, " 80 × 160 = 12800
" Jan. 1, 1869, 100 × 212 = 21200
$935)$82565(88.3

The equated time is 88 days forward from June 3, 1868, which is August 30, 1868.

If the accounts were settled December 1, 1868, besides the amount of the bill, $935, there should be paid the interest on the same from the equated date of maturity, August 30, to December 1, 1868, or 93 days, which, at 6%, would be $14.49, making the whole sum to be paid December 1, 1868, $949.49.

Page 219.

8.	*Dr.*		*Cr.*
Due June 5, 1870,	$1285	Due July 1, 1870,	$1000

Debits, $1285 Difference in time, 26 days.
Credits, 1000
Balance, $285

$$\frac{\$1285 \times 26}{\$285} = 117.2$$

Balance was due 117 days *prior* to July 1, 1870, which was March 6, 1870. See Comp. Arith., p. 218, Ex. 7.

9.	*Dr.*		*Cr.*
Due July 12, 1870,	$450	Due Sept. 1, 1870,	$800

Credits, $800 Difference in time, 51 days.
Debits, 450
Balance, $350

$$\frac{\$450 \times 51}{\$350} = 65.6$$

Balance was due and began to draw interest 66 days *after* September 1, 1870, which was November 6, 1870.

Page 219. (Continued.)

Debits.

10.	Due	Oct.	10,	1870,	$120	
	"	Oct.	30,	"	$450 \times 20 =$	$ 9000
	"	Nov.	30,	"	$380 \times 51 =$	19380
	"	Dec.	9,	"	$560 \times 60 =$	33600
	"	Dec.	30,	"	$400 \times 81 =$	32400
					$1910	)$94380(49.4

Debits due 49 days after October 10, 1870, which was November 28, 1870.

Credits.

Due	Nov.	20,	1870,	$350	
"	Dec.	25,	"	$250 \times 35 =$	$ 8750
"	Jan.	1,	1871,	$750 \times 42 =$	31500
				$1350	)$40250(29.8

Credits due 30 days after November 20, 1870, which was December 20, 1870.

As equated, the account stands thus:

	Dr.		*Cr.*
Due Nov. 28, 1870,	$1910	Due Dec. 20, 1870,	$1350

Debits,	$1910	Difference in time, 22 days.
Credits,	1350	
Balance,	$560	

$$\frac{\$1910 \times 22}{\$560} = 75$$

Balance was due and began to draw interest 75 days *prior* to December 20, 1870, which was October 6, 1870.

PROPORTION.

Page 227.

35. 4 lb. 6 oz. = 4.375 lb.; $17\frac{1}{4}$ lb. = 17.25 lb.;
4.375 lb. : 17.25 lb. : : $1.75 : Ans.
17.25
4.375)$30.1875
$6.90, Ans.

36. 1 min. : 480 min. : : 75 : Ans.
75 × 480 ÷ 1 = 3600, Ans.

39. 6 wk. : 52 wk. : 24 bbl. : Ans.
52
6)1248 bbl.
208 bbl., Ans.

41. 3 ft. : 120 ft. : : 5 ft. shadow : Ans.
120
3)600 ft.
200 ft., Ans.

42. 12 ft. shadow : 90 ft. shadow : : 20 ft. : Ans.
90
12)1800 ft.
150 ft., Ans.

Page 228.

43. $\frac{5}{9} : \frac{3}{4} :: \$4500$: Ans.
$\$4500 \times \frac{3}{4} \div \frac{5}{9} = \$6075$, Ans.

46. 5 peaches : 60 peaches : : 8 apples : Ans.
8 apples × 60 ÷ 5 = 96 apples, Ans.

50. 5 da. : 12 da. : : 15 men : Ans.
12
5)180 men.
36 men, Ans.

Page 229.

53. $(15 + 17) : 15 :: \$9600$: share of younger.

$$\begin{array}{r} 15 \\ \hline 32)\overline{\$144000} \\ \hline \$4500, \end{array} \text{ share of younger.}$$

$(15 + 17) : 17 :: \$9600$: share of elder.

$$\begin{array}{r} 17 \\ \hline 32)\overline{\$163200} \\ \hline \$5100, \end{array} \text{ share of elder.}$$

Or, after the share of one has been obtained as above, the share of the other may be found by subtraction.

54. $(2 + 3) : 2 ::$ 150 miles : distance traveled by 1st.
150 miles $\times 2 \div 3 =$ 60 miles, distance by 1st.
150 miles — 60 miles = 90 miles, distance by 2d.

Page 233.

22. $\left.\begin{array}{l} 24 : 8 \\ 16 : 4 \end{array}\right\} :: 1500 : \text{Ans.}$

$1500 \times 8 \times 4 \div 24 \div 16 = 125$, slabs.

25. $\left.\begin{array}{lcl} 40 \text{ ft.} & : & 180 \text{ ft.} \\ 14 \text{ ft.} & : & 21 \text{ ft.} \\ 1\frac{1}{2} \text{ ft.} & : & 1\frac{1}{4} \text{ ft.} \end{array}\right\} :: \$120 : \text{Ans.}, = \$675.$

26. $\left.\begin{array}{lcl} 72 \text{ rd.} & : & 120 \text{ rd.} \\ 5 \text{ ft.} & : & 6 \text{ ft.} \\ 2 \text{ ft.} & : & 1\frac{1}{2} \text{ ft.} \\ 9 \text{ da.} & : & 12 \text{ da.} \end{array}\right\} :: 4 \text{ men} : \text{Ans.}, = 8 \text{ men.}$

27. $\left.\begin{array}{lcl} 6 \text{ men} & : & 16 \text{ men} \\ 50 \text{ ft.} & : & 45 \text{ ft.} \\ 36 \text{ ft.} & : & 25 \text{ ft.} \\ 8 \text{ ft.} & : & 6 \text{ ft.} \\ 10 \text{ hr.} & : & 8 \text{ hr.} \end{array}\right\} :: 10 \text{ days} : \text{Ans.}, = 10 \text{ days.}$

Page 233. (Continued.)

28.

$$\left.\begin{array}{rlcrl} 40 & \text{rd.} & : & 15 & \text{rd.} \\ 6 & \text{ft.} & : & 4\frac{1}{2} & \text{ft.} \\ 3 & \text{ft.} & : & 2 & \text{ft.} \\ 12 & \text{da.} & : & 9 & \text{da.} \\ 6 & \text{hr.} & : & 8 & \text{hr.} \end{array}\right\} :: 32 \text{ men} : \text{Ans.}, = 6 \text{ men.}$$

PARTNERSHIP.

Page 236.

4. $1 + \frac{3}{4} + \frac{3}{4} \times \frac{2}{3} = 2\frac{1}{4}$; the whole stock was $2\frac{1}{4}$ times A's stock.

$2\frac{1}{4} : 1 ::$ \$6300 : \$2800, A's profits.
$2\frac{1}{4} : \frac{3}{4} ::$ \$6300 : \$2100, B's "
$2\frac{1}{4} : \frac{1}{2} ::$ \$6300 : \$1400, C's "

5. \$10000 — \$2300 = \$7700, net profits;

7 : 4 :: \$7700 : \$4400, share of first;
7 : 3 :: \$7700 : \$3300, " " second.

Or, \$7700 × $\frac{4}{7}$ = \$4400; \$7700 × $\frac{3}{7}$ = \$3300.

6. \$4000 + \$6400 + \$5600 = \$16000, entire capital.

\$16000 : \$4000 :: \$3200 : A's share = \$ 800;
\$16000 : \$6400 :: \$3200 : B's " = \$1280;
\$16000 : \$5600 :: \$3200 : C's " = \$1120.

Or, $\frac{4000}{16000} = \frac{1}{4}$ = A's part of capital.

$\frac{6400}{16000} = \frac{2}{5}$ = B's " "

$\frac{5600}{16000} = \frac{7}{20}$ = C's " "

Page 236. (Continued.)

$\$3200 \times \frac{1}{4} = \$800$, A's gain.
$\$3200 \times \frac{2}{5} = \$1280$, B's “
$\$3200 \times \frac{7}{20} = \$1120$, C's “

$\$2000 \times \frac{1}{4} = \$500$, A's share of proceeds.
$\$2000 \times \frac{2}{5} = \$800$, B's “ “ “
$\$2000 \times \frac{7}{20} = \$700$, C's “ “ “

Page 238.

8. $\$6000 \times 2 = \$12000$
$\$4000 \times 2 = \$8000$
$\$20000$, A's capital for 1 year.

$\$8000 \times 2 = \$16000$
$\$10000 \times 2 = \$20000$
$\$36000$, B's capital for 1 year.

$\$20000 + \$36000 = \$56000$, entire capital for 1 year.
$\$56000 : \$20000 :: \$8890$: A's share $= \$3175$;
$\$56000 : \$36000 :: \$8890$: B's share $= \$5715$.

9. $\$3500 \times 1 = \$3500$
$\$2500 \times 2 = \$5000$
$\$8500$, A's capital for 1 year.

$\$4500 \times 1 = \$4500$
$\$3500 \times 2 = \$7000$
$\$11500$, B's capital for 1 year.

$\$2500 \times 2 = \$5000$, C's capital for 1 year.

$\$8500 + \$11500 + \$5000 = \$25000$, entire capital for 1 year.

$\$25000 : \$8500 :: \$5000$: A's share $= \$1700$;
$\$25000 : \$11500 :: \$5000$: B's share $= \$2300$;
$\$25000 : \$5000 :: \$5000$: C's share $= \$1000$.

Page 238. (Continued.)

10. $5000 × 2 = $10000
$4000 × 2 = $ 8000
$18000, A's capital for 1 year.

$5000 × 2 = $10000
$3000 × 2 = $ 6000
$16000, B's capital for 1 year.

$5000 × 2 = $10000
$2000 × 2 = $ 4000
$14000, C's capital for 1 year.

$18000 + $16000 + $14000 = $48000, entire capital for 1 year.

$48000 : $18000 :: $3600 : A's share of loss = $1350;
$48000 : $16000 :: $3600 : B's share of loss = $1200;
$48000 : $14000 :: $3600 : C's share of loss = $1050.

PROBLEMS FOR ANALYSIS.

Page 239.

7. A yard of carpeting, $\frac{3}{4}$ of a yard wide, contains $\frac{3}{4}$ of a square yard, and if 40 yards of it will cover a floor, the floor contains 40 times $\frac{3}{4}$, or $\frac{120}{4}$, square yards; and since a yard of matting, $1\frac{1}{4}$ yards wide, contains $\frac{5}{4}$ of a square yard, it will take as many yards of the matting to cover a floor of equal size as $\frac{5}{4}$ is contained times in $\frac{120}{4}$, which is 24; it will take 24 yards of matting.

Or, if 40 yards of carpeting, $\frac{3}{4}$ of a yard wide, will cover a floor, it will take 3 times 40 yards, or 120 yards of carpeting, $\frac{1}{4}$ of a yard wide, and if it takes 120 yards of carpet-

Page 239. (Continued.)

ing, $\frac{1}{4}$ of a yard wide, to cover a floor, it will take $\frac{1}{5}$ of 120 yards, or 24 yards, of matting, $\frac{5}{4}$ of a yard wide, to cover a floor of equal size.

8. If one man travels 5 miles an hour and the other 7 miles, the latter gains 2 miles an hour, and it will take him as many hours to overtake the former as 2 miles are contained times in 60 miles, which is 30.

10. Since the height of the steeple is $\frac{200}{150}$, or $\frac{4}{3}$, of the length of its shadow, the height of the pole is $\frac{4}{3}$ of the length of its shadow, or $\frac{4}{3}$ of 80 feet, which is 106$\frac{2}{3}$ feet.

12. If 8 men can do a piece of work in 15 days, it will take 1 man 8 times 15 days, or 120 days, and it will take as many men to do it in 10 days as 10 days are contained times in 120 days, which is 12; it will take 12 men to do the work in 10 days.

Page 240.

16. If it require 12 days, of 10 hours each, to do the work, it requires 12 times 10 hours, or 120 hours, and as many days, of 8 hours each, as 8 hours are contained times in 120 hours, which is 15.

18. If 8 men can do $\frac{3}{4}$ of the work in 3 days, they can do $\frac{1}{3}$ of $\frac{3}{4}$, or $\frac{1}{4}$ of it, in a day, and 1 man can do $\frac{1}{8}$ of $\frac{1}{4}$, or $\frac{1}{32}$ of it in a day, and 4 men can do 4 times $\frac{1}{32}$, or $\frac{1}{8}$ of it in a day; if 4 men can do $\frac{1}{8}$ of the work in a day, it will take them as many days to do the entire work as $\frac{1}{8}$ is contained times in $\frac{8}{8}$, which is 8.

Or, if 8 men can do $\frac{3}{4}$ of the work in 3 days, they can do $\frac{1}{4}$ of it in 1 day, and if 8 men can do $\frac{1}{4}$ of it, 4 men

Page 240. (Continued.)

can do $\frac{1}{2}$ of $\frac{1}{4}$, or $\frac{1}{8}$, of it in 1 day, and if 4 men can do $\frac{1}{8}$ of the work in a day, it will take them 8 days to do the entire work.

20. If 6 men can mow 30 acres in 3 days, 1 man can mow $\frac{1}{6}$ of 30 acres, or 5 acres, in 3 days, and $\frac{1}{3}$ of 5 acres, or $\frac{5}{3}$ of an acre, in 1 day; and if 1 man can mow $\frac{5}{3}$ of an acre in 1 day, 9 men can mow 9 times $\frac{5}{3}$ of an acre, or 15 acres, in 1 day, and 5 times 15 acres, or 75 acres, in 5 days.

23. If the interest of \$50 for 9 months is \$6, the interest of \$50 for 1 month is $\frac{1}{9}$ of \$6, or $\$\frac{2}{3}$, and the interest of \$150 for 1 month would be 3 times $\$\frac{2}{3}$, which is \$2, and for 18 months, 18 times \$2, which is \$36.

26. If $\frac{2}{5}$ of the shorter part equals $\frac{2}{7}$ of the longer part, $\frac{1}{5}$ of the shorter equals $\frac{1}{7}$ of the longer, and $\frac{5}{5}$, or the whole of the shorter, equals $\frac{5}{7}$ of the longer, and hence $\frac{7}{7}+\frac{5}{7}$, or $\frac{12}{7}$, of the longer part equals 120 feet, and $\frac{1}{7}$ of it equals $\frac{1}{12}$ of 120 feet, or 10 feet, and $\frac{7}{7}$ of it equals 70 feet; if the longer part is 70 feet, the shorter is $\frac{5}{7}$ of 70 feet, which is 50 feet.

27. If $\frac{2}{3}$ of the time past noon equaled the time to midnight, $\frac{3}{3}$ plus $\frac{2}{3}$, or $\frac{5}{3}$, of the time past noon equaled the whole time from noon to midnight, or 12 hours; if $\frac{5}{3}$ of the time past noon was 12 hours, $\frac{1}{3}$ of it was $\frac{1}{5}$ of 12 hours, or $2\frac{2}{5}$ hours, and $\frac{3}{3}$, or the time past noon, was 3 times $2\frac{2}{5}$ hours, which is $7\frac{1}{5}$ hours, or 7 hours and 12 minutes: the time was 12 minutes past 7 P. M.

28. If $\frac{1}{3}$ of the time past midnight equaled the time to noon, $\frac{4}{3}$ of the time past midnight equaled the whole time from midnight to noon, or 12 hours; if $\frac{4}{3}$ of the time past midnight was 12 hours, $\frac{3}{3}$, or the time, was 9 hours: the time was 9 A. M.

Page 241.

29. If $\frac{3}{4}$ of the time past noon equals $\frac{3}{8}$ of the time to midnight, $\frac{1}{4}$ time past noon is $\frac{1}{8}$ of the time to midnight, and $\frac{4}{4}$, or the time past noon, is $\frac{4}{8}$, or $\frac{1}{2}$, of the time to midnight; and hence $\frac{1}{2}$ plus $\frac{2}{2}$, or $\frac{3}{2}$, of the time to midnight is 12 hours, and the whole time to midnight is 8 hours, and the time past noon is 4 hours: the time is 4 P. M.

32. If the man's present age less 10 years equals $\frac{7}{9}$ of his present age, 10 years must be $\frac{2}{9}$ of his present age, and hence $\frac{1}{9}$ is 5 years, and $\frac{9}{9}$, or his present age, is 45 years.

34. If A's age 10 years ago was $\frac{3}{4}$ of B's age 10 years ago, 10 years hence A's age will be $\frac{3}{4}$ of B's age 10 *years ago* plus 20 years; by the second condition, A's age 10 years hence will equal $\frac{5}{6}$ of B's age 10 years hence, or $\frac{5}{6}$ of B's age 10 *years ago* plus $\frac{5}{6}$ of 20 years. Comparing the two values of A's age 10 years hence, as found above, we have $\frac{3}{4}$ of B's age 10 years ago plus 20 years equal to $\frac{5}{6}$ of B's age 10 years ago plus $16\frac{2}{3}$ years, and hence the difference between $\frac{5}{6}$ and $\frac{3}{4}$ of B's age 10 years ago, or $\frac{1}{12}$ of his age, is equal to the difference between 20 years and $16\frac{2}{3}$ years, which is $3\frac{1}{3}$ years, and if $\frac{1}{12}$ of B's age 10 years ago was $3\frac{1}{3}$ years, his age then was 40 years, and his present age is 50 years. A's age 10 years ago was $\frac{3}{4}$ of 40 years, or 30 years, and his present age is 40 years.

Or, if A's age 10 years ago was $\frac{3}{4}$ of B's age at that time, 10 years hence A's age will be $\frac{3}{4}$ of B's age 10 years ago + 20 years; but A's age 10 years hence will equal $\frac{5}{6}$ of B's age 10 years hence, and thus $\frac{5}{6}$ of B's age 10 years hence will equal $\frac{3}{4}$ of B's age ten years ago + 20 years; and $\frac{1}{6}$ of B's age 10 years hence is $\frac{3}{20}$ of B's age 10 years ago + 4 years, and $\frac{6}{6}$, or B's age 10 years hence, is $\frac{9}{10}$ of

Page 241. (Continued.)

B's age 10 years ago + 24 years. Thus the difference between B's age 10 years hence and $\frac{9}{10}$ of his age 10 years ago is 24 years, and hence $\frac{1}{10}$ of B's age 10 years ago was 24 years less 20 years, or 4 years, and his age at that time was 40 years, and his present age is 50 years. A's age 10 years ago was $\frac{3}{4}$ of 40 years, or 30 years, and his present age is 40 years.

The following solution is briefer than either of the above, and may be preferred:

Suppose that A's age 10 years ago was 3 years and B's age 4 years, A's being $\frac{3}{4}$ of B's; in 2 years A's age was 5 years and B's 6 years, and A's was then $\frac{5}{6}$ of B's; but, by the conditions of the problem, A's age will be $\frac{5}{6}$ of B's in 20 years, or in 10 times 2 years, and hence A's age 10 years ago was 10 times 3 years, or 30 years, and his present age is 40 years, and B's age 10 years ago was 10 times 4 years, or 40 years, and his present age is 50 years.

NOTE.—It will be noticed that the multiplier of 3 years and 4 years, is found by dividing 20 by the number (2) that must be added to both terms of $\frac{3}{4}$ to make $\frac{5}{6}$. The so-called "age problems" may all be solved in this simple manner.

35. If the wife's age at marriage was $\frac{3}{5}$ of the husband's age at marriage, 10 years after marriage her age was $\frac{3}{5}$ of the husband's age at marriage + 10 years; by the second condition, 10 years after marriage her age was $\frac{7}{10}$ of the husband's age 10 years after marriage, or $\frac{7}{10}$ of his age at marriage + $\frac{7}{10}$ of 10 years, or 7 years. Comparing these two values of the wife's age 10 years after marriage, it is seen that $\frac{3}{5}$ of the husband's age at marriage + 10 years was equal to $\frac{7}{10}$ of his age at marriage + 7 years, and hence $\frac{7}{10}$

Page 241. (Continued.)

— $\frac{3}{5}$, or $\frac{1}{10}$, of his age at marriage was 10 years — 7 years, or 3 years, and his age at marriage was 30 years. The wife's age at marriage was $\frac{3}{5}$ of 30 years, or 18 years.

Or, by the conditions, the wife's age 10 years after marriage was $\frac{3}{5}$ of the husband's age at marriage + 10 years, and this was $\frac{7}{10}$ of the husband's age 10 years after marriage; whence $\frac{1}{10}$ the husband's age 10 years after marriage is $\frac{3}{35}$ of his age at marriage + $1\frac{3}{7}$ years; and $\frac{10}{10}$, or his age 10 years after marriage, is $\frac{6}{7}$ his age at marriage + $14\frac{2}{7}$ years. Thus $14\frac{2}{7}$ years added to *six sevenths* of his age at marriage, or 10 years added to his whole age at marriage give the same result, namely, his age 10 years after marriage; from this it is seen that $\frac{1}{7}$ his age at marriage is $4\frac{2}{7}$ years, whence his whole age at marriage was 30 years. The wife's age at marriage was $\frac{3}{5}$ of 30, or 18 years.

Or, suppose that the wife's age at marriage was 3 years and her husband's age 5 years, her age being $\frac{3}{5}$ of his age; in $1\frac{2}{3}$ years after marriage her age was $4\frac{2}{3}$ years and his $6\frac{2}{3}$ years, and her age was then $\frac{7}{10}$ of his ($4\frac{2}{3} \div 6\frac{2}{3} = \frac{7}{10}$); but by the conditions of the problem, her age was $\frac{7}{10}$ of his in 10 years, or in 6 times $1\frac{2}{3}$ years, and hence her age at marriage was 6 times 3 years, or 18 years, and his age was 6 times 5 years, or 30 years.

37. From the conditions, B's age is 20 years less than twice A's age, and C's age is 10 years less than twice A's; thus the sum of A's, B's, and C's ages is 5 times A's age less 30 years, and this equals 120 years; whence 5 times A's age is 120 plus 30, or 150 years, and his age is $\frac{1}{5}$ of 150 years, which is 30 years. B's age equals twice 30 years minus 20 years, or 40 years, and C's age equals twice 30 years minus 10 years, or 50 years.

Page 241. (Continued.)

38. If $\frac{1}{3}$ the cost of horse was $\frac{1}{5}$ the cost of carriage + \$33, then the whole cost of horse was 3 times $\frac{1}{5}$, or $\frac{3}{5}$, of the cost of carriage + \$99; and hence $\frac{5}{5}$ the cost of carriage + $\frac{3}{5}$ its cost + \$99 equals \$275, the whole cost of horse and carriage; if $\frac{5}{5} + \frac{3}{5}$, or $\frac{8}{5}$, the cost of carriage equals \$275 — \$99, or \$176, $\frac{5}{5}$, or the cost of carriage, equals \$110; $\frac{3}{5}$ of \$110 + \$99 = \$66 + \$99 = \$165, cost of horse.

39. From the conditions, the cost of horse, plus $\frac{1}{6}$ the cost of horse, plus $\frac{1}{2}$ of $\frac{1}{6}$, or $\frac{1}{12}$, of the cost of horse, equals \$150; that is, $\frac{15}{12}$, or $\frac{5}{4}$, of the cost of the horse equals \$150, and hence $\frac{4}{4}$, or the cost of horse, equals \$120; $\frac{1}{6}$ of \$120 = \$20, cost of saddle; $\frac{1}{12}$ of \$120 = \$10, the cost of bridle.

40. From the conditions, the man's wages, plus $\frac{1}{2}$ the man's wages, plus $\frac{1}{2}$ of $\frac{1}{2}$, or $\frac{1}{4}$, of the man's wages, equals \$140; that is, $\frac{7}{4}$ the man's wages equals \$140, and hence $\frac{4}{4}$, or the man's wages, equals \$80; $\frac{1}{2}$ of \$80 equals \$40, elder son's wages; $\frac{1}{4}$ of \$80 equals \$20, younger son's wages.

Or, by the conditions, the younger son's wages, plus twice his wages, plus four times his wages, or seven times his wages, equals \$140, and hence his wages equals \$20, etc.

Page 242.

43. If 4 sheep eat as much as 1 cow, 40 sheep will eat as much as 10 cows, and A should pay the same as he would have paid if he had put in 10 cows; hence A's share is $\frac{10}{18}$ of \$72, which is \$40; and B's share is $\frac{8}{18}$ of \$72, which is \$32.

44. Five men in 4 days did 20 days' work, and five men in 6 days, 30 days' work, and hence it took 20 + 30, or 50,

Page 242. (Continued.)

days' work to build the wall. A ought to receive $\frac{20}{50}$, or $\frac{2}{5}$, of \$300, which is \$120; and B $\frac{30}{50}$, or $\frac{3}{5}$, of \$300, which is \$180. Therefore A should receive \$120 and B \$180.

46. The man and boy can both do $\frac{1}{5} + \frac{1}{8}$, which is $\frac{13}{40}$, of the work in one day; it will take both as many days to do the work as $\frac{13}{40}$ is contained times in $\frac{40}{40}$, which is $3\frac{1}{13}$ times, or $3\frac{1}{13}$ days.

48. Since A can do the work in 6 days and B in 8 days, A can do $\frac{1}{6}$ of the work and B $\frac{1}{8}$ of it in 1 day and in 3 days they would do $\frac{3}{6}$ plus $\frac{3}{8}$ of the work, which is $\frac{7}{8}$ of it, leaving $\frac{1}{8}$ of it undone; and since B can do $\frac{1}{8}$ of it in a day, it will take him 1 day to complete the work.

50. Since the man can do $\frac{1}{5}$ of the work in 1 day, in 2 days he did $\frac{2}{5}$ of it, leaving $\frac{3}{5}$ to be done by the man and boy together; since the man can do $\frac{1}{5}$ and the boy $\frac{1}{8}$ of the work in a day, they can together do $\frac{1}{5}$ plus $\frac{1}{8}$, or $\frac{13}{40}$, of it, and it will take them as many days to complete the work as $\frac{13}{40}$ is contained times in $\frac{3}{5}$, or $\frac{24}{40}$, which is $1\frac{11}{13}$; it will take them $1\frac{11}{13}$ days.

51. Since A and B can do $\frac{1}{10}$ of the work in a day, and A, B, and C $\frac{1}{8}$ of it, the difference between $\frac{1}{8}$ and $\frac{1}{10}$, which is $\frac{1}{40}$, is the part of the work C can do in one day, and hence he can do the work in 40 days.

52. Since A and B can do $\frac{1}{8}$ of the work in one day and A can do twice as much as B, A can do $\frac{2}{3}$ of $\frac{1}{8}$ the work in one day, and B $\frac{1}{3}$ of $\frac{1}{8}$ of it, which is $\frac{1}{24}$ of it, and hence it will take B 24 days to do the work.

Page 242. (Continued.)

53. Since A can make a fence in $\frac{1}{4}$ of a month, he can make 4 such fences in one month; B can make 5 and C 6 such fences in one month, and A, B, and C can together make 4, plus 5, plus 6, or 15, such fences in one month, and hence it would take them $\frac{1}{15}$ of a month to make the fence.

NOTE.—The problem may be solved, as previous ones, by making $\frac{1}{4}$ of a month equal to 6 days, $\frac{1}{5}$ equal to $4\frac{4}{5}$ days, and $\frac{1}{6}$ equal to 4 days.

55. Since A, B, and C can do $\frac{1}{4}$ of the work in one day, and A and C $\frac{1}{8}$ of it, B can do $\frac{1}{4}$ minus $\frac{1}{8}$, or $\frac{1}{8}$, of it in one day; and since A, B, and C can do $\frac{1}{4}$, and B and C $\frac{1}{6}$ of it in one day, A can do $\frac{1}{4}$ minus $\frac{1}{6}$, or $\frac{1}{12}$, of it in one day; if A can do $\frac{1}{12}$ and B $\frac{1}{8}$ of the work in one day, they can both do $\frac{1}{8}+\frac{1}{12}$, or $\frac{5}{24}$, of it in one day, and hence it will take them as many days to do the work as $\frac{5}{24}$ is contained times in $\frac{24}{24}$, which is $4\frac{4}{5}$: A and B can do the work in $4\frac{4}{5}$ days.

Page 243.

56. If $\frac{2}{5}$ of what A did equals $\frac{4}{5}$ of what B did, $\frac{1}{5}$ of what A did equals $\frac{2}{5}$ of what B did, and $\frac{5}{5}$, or A's entire work, equals 5 times $\frac{2}{5}$ of B's work, or 2 times B's work; therefore A should receive twice \$18, which is \$36.

57. If a man spent $\frac{3}{4}$ of his money, he had $\frac{1}{4}$ left; if he then earned $\frac{1}{2}$ of $\frac{3}{4}$, or $\frac{3}{8}$, he had $\frac{1}{4}+\frac{3}{8}$, or $\frac{5}{8}$, of his money, which is $\frac{3}{8}$ less than he had at first; if $\frac{3}{8}$ of his money at first was \$21, $\frac{1}{8}$ was \$7, and $\frac{8}{8}$, or amount at first, \$56.

58. The minute-hand passes over 60 minute spaces in one hour and the hour-hand over 5, and in one minute the minute-hand passes over 1 space and the hour-hand over $\frac{1}{12}$

Page 243. (Continued.)

of a space, and hence the minute-hand gains on the hour-hand $\frac{11}{12}$ of a space each minute. At one o'clock the minute-hand is 5 spaces behind the hour-hand, and it will take as many minutes to overtake it as $\frac{11}{12}$ of a space is contained times in 5 spaces, which is $5\frac{5}{11}$: hence the hands will be together $5\frac{5}{11}$ minutes past one.

Or, since the minute-hand passes over 12 spaces while the hour-hand passes over 1, it gains 11 spaces in passing over 12, and hence the distance passed over by the minute-hand is $\frac{12}{11}$ of the distance gained by it, and so to gain the 5 minute spaces, which separate the two hands at one o'clock, the minute-hand must pass over $\frac{12}{11}$ of 5 spaces, which is $5\frac{5}{11}$ spaces; and since this requires $5\frac{5}{11}$ minutes, the two hands will be together at $5\frac{5}{11}$ minutes past one o'clock.

59. At two o'clock the minute-hand is 10 spaces behind the hour-hand, and it will take as many minutes for it to overtake the hour-hand as $\frac{11}{12}$ of a space is contained times in 10 spaces, which is $10\frac{10}{11}$: the two hands will be together at $10\frac{10}{11}$ minutes past two o'clock. At four o'clock the two hands are 20 spaces apart, and it will take as many minutes for the minute-hand to overtake the hour-hand as $\frac{11}{12}$ of a space is contained times in 20 spaces, which is $21\frac{9}{11}$, and the two hands will be together at $21\frac{9}{11}$ minutes past four o'clock.

Or, since the minute-hand passes over $\frac{12}{11}$ of the distance gained, at two o'clock it must pass over $\frac{12}{11}$ of 10 spaces, or $10\frac{10}{11}$ spaces, to overtake the hour-hand, and, at four o'clock, it must pass over $\frac{12}{11}$ of 20 spaces, or $21\frac{9}{11}$ spaces; hence the two hands are together at $10\frac{10}{11}$ minutes past two, and at $21\frac{9}{11}$ minutes past four.

Page 243. (Continued.)

60. Since $\frac{2}{3}$ of the value of the second watch equals the value of the first watch plus $20, $\frac{3}{3}$, or the value of the second watch, equals $\frac{3}{2}$ of the value of the first watch plus $30; by the second condition, the value of the second watch plus $20 (value of chain) equals $2\frac{3}{4}$ times the value of the first watch, and hence the value of the second watch equals $2\frac{3}{4}$ times the value of the first watch minus $20. Since the two values of the second watch, thus found, are equal to each other, we have $2\frac{3}{4}$ times the value of first watch minus $20 equal to $1\frac{1}{2}$ times the value of first watch plus $30; and hence $2\frac{3}{4}$ times the value of first watch minus $1\frac{1}{2}$ times its value, or $\frac{5}{4}$ of its value, equals $50, and $\frac{1}{4}$ its value is $10, and $\frac{4}{4}$, or its value, is $40. Since $\frac{2}{3}$ of the value of second watch equals $40 + $20, or $60, its whole value is $90.

61. $\frac{10}{10} + \frac{7}{10} = \frac{17}{10}$;
$14535 ÷ $\frac{17}{10}$ = $8550, elder's share;
$14535 — $8550 = $5985, younger's share.

62. $\frac{3}{4}$ of younger's ÷ $\frac{2}{3}$ = $\frac{9}{8}$ of younger's = elder's share;
$\frac{9}{8}$ of younger's — $\frac{8}{8}$ of younger's = $\frac{1}{8}$ of younger's = $362;

$362 × 8 = $2896, younger's share;
$2896 × $\frac{9}{8}$ = $3258, elder's share.

63. (14 + 18) : 14 : : $27520 : share of younger = $12040;
(14 + 18) : 18 : : $27520 : elder's share = $15480.

Or, $27520 × $\frac{7}{16}$ = $12040, share of younger;
$27520 × $\frac{9}{16}$ = $15480, " " elder.

Page 243. (Continued.)

64. $\frac{1}{5}$ of the cost of the larger farm equals $\frac{3}{10}$ of the cost of the smaller, and $\frac{5}{5}$, or the whole cost, of the larger farm, equals $\frac{15}{10}$, or $\frac{3}{2}$, the cost of the smaller; and hence both farms cost $\frac{5}{2}$ as much as the smaller.

$8100 ÷ $\frac{5}{2}$ = $3240, cost of smaller farm;
$8100 — $3240 = $4860, cost of larger farm.

65. A earns $15.50 as often as both earn $27.90; while they both earn $697.50, A earns as many times $15.50 as $27.90 is contained times in $697.50, which is 25.

$15.50 × 25 = $387.50, the amount A earns;
$12.40 × 25 = $310.00, the amount B earns.

66. 12½ ft. : 9½ ft. :: 9500 times : Ans., = 7220 times.

Or = 9½ ft. × 9500 = 90250 ft., distance;
90250 ÷ 12½ = 7220, revolutions of each hind wheel.

Page 244.

67. 33 in. × 13200 = 435600 in., distance;
435600 in. ÷ 22 in. = 19800, steps, 1 ft 10 in. each.

68. $10.80 ÷ 75 = $.144, interest on $1,
$89.28 ÷ $.144 = 620;
$1 × 620 = $620, principal required.

69. $118.75 ÷ 475 = $.25, interest on $1;
$.25 × 850 = $212.50, interest required.

70. $120.50 ÷ 5 × 12 = $289.20.

Or, 12 ÷ 5 = 2$\frac{2}{5}$; $120.50 × 2$\frac{2}{5}$ = $289.20.

Page 244. (Continued.)

71. $\$315 \div 90 = 3.50$, gain on 1 share;
$\$3.50 \times 245 = \$857.50$, gain on 245 shares.

72. $\$2347.50 \div 15 = \$156.50$, yield at 1%;
$\$156.50 \times 24 = \$3756$, yield at 24%.

73. $\$.30 \times 3050 = \$915$, value of 1st butter sold;
$\$45.75 \div \$915 = .05 = 5\%$, rate of commission;
$\$.35 \times 7500 = \$2625$, value of 2d butter sold;
$\$2625 \times .05 = \$131.25$, commission.

74. $\$3226 \div \$40325 = .08 = 8\%$, rate;
$\$70680 \times .08 = \$5654.40$, dividend.

75.
$$\left.\begin{array}{lcl} 6 \text{ ranks} & : & 15 \text{ ranks} \\ 60 \text{ ft.} & : & 45 \text{ ft.} \\ 6 \text{ ft.} & : & 9 \text{ ft.} \end{array}\right\} :: \$337.50 : \text{Ans.}, = \$949.218.$$

Or, $\$337.50 \div 6 \div 60 \div 6 \times 15 \times 45 \times 9 = \$949.218$.

76.
$$\left.\begin{array}{lcl} 40 \text{ ft.} & : & 36 \text{ ft.} \\ 27 \text{ ft.} & : & 30 \text{ ft.} \\ 4 \text{ ft.} & : & 5 \text{ ft.} \end{array}\right\} :: \$110 : \text{Ans.}, = \$137.50.$$

77.
$$\left.\begin{array}{lcl} 30 \text{ men} & : & 45 \text{ men} \\ 9 \text{ hr.} & : & 8 \text{ hr.} \end{array}\right\} :: 15 \text{ da.} : \text{Ans.}, = 20 \text{ da.}$$

78. 45 cords $\div 5 \div 6 = 1\frac{1}{2}$ cords, cut by 1 man in 1 day;
$1\frac{1}{2}$ cords $\times 8 \times 15 = 180$ cords.

Page 245.

79. 10 h. $\times 15 \times 4 = 600$ h., time 1 man requires;
600 h. $\div 8$ h. $= 75$, days 1 man requires;
75 days $\div 5 = 15$ days.

Page 245. (Continued.)

80. $\frac{5}{8}$ B's + $\frac{8}{8}$ B's = $\frac{13}{8}$ B's = A's and B's capital;
$\frac{5}{8} \div \frac{13}{8} = \frac{5}{13}$, A's part of profits;
$\frac{8}{8} \div \frac{13}{8} = \frac{8}{13}$, B's “ “ “

$3250 × $\frac{5}{13}$ = $1250, A's share;
$3250 × $\frac{8}{13}$ = $2000, B's “

81. $\frac{5}{6}$ B's × $\frac{5}{3}$ = $\frac{25}{18}$ B's = A's capital;
$\frac{25}{18}$ B's + $\frac{18}{18}$ B's = $\frac{43}{18}$ B's = A's and B's capital;

$2150 × $\frac{25}{43}$ = $1250, A's share of loss;
$2150 × $\frac{18}{43}$ = $ 900, B's “ “ “

82. A's capital = twice B's; B's = $\frac{1}{2}$ A's;
A's “ = 3 times C's; C's = $\frac{1}{3}$ A's;
A's + $\frac{1}{2}$ A's + $\frac{1}{3}$ A's = $1\frac{5}{6}$ A's = entire capital.

1 A's ÷ $1\frac{5}{6}$ A's = $\frac{6}{11}$, A's part of loss;
$\frac{1}{2}$ A's ÷ $1\frac{5}{6}$ A's = $\frac{3}{11}$, B's “ “ “
$\frac{1}{3}$ A's ÷ $1\frac{5}{6}$ A's = $\frac{2}{11}$, C's “ “ “

$4675 × $\frac{6}{11}$ = $2550, A's share;
$4675 × $\frac{3}{11}$ = $1275, B's “
$4675 × $\frac{2}{11}$ = $ 850, C's “

Or, $\frac{11}{6}$: $\frac{6}{6}$ :: $4675 : A's share = $2550;
$\frac{11}{6}$: $\frac{1}{2}$ :: $4675 : B's share = $1275;
$\frac{11}{6}$: $\frac{1}{3}$ :: $4675 : C's share = $ 850.

83. $5460 ÷ 2 = $2730, gain each year;
$2730 × $\frac{2}{3}$ = $1820, A's gain 1st year;
$2730 × $\frac{1}{4}$ = $ 682.50, “ “ 2d “
$2502.50, A's whole gain.

$2730 × $\frac{1}{3}$ = $ 910, B's gain 1st year;
$2730 × $\frac{3}{4}$ = $2047.50, “ “ 2d “
$2957.50, B's whole gain.

Page 245. (Continued.)

Or, \$5460 — \$2502.50 = \$2957.50, B's gain.

Or thus: $\frac{2}{3} + \frac{1}{4} = \frac{11}{12}$, A's capital for 1 year;
$\frac{1}{3} + \frac{3}{4} = \frac{13}{12}$, B's " " " "

$\frac{11}{12} + \frac{13}{12} = \frac{24}{12}$, entire capital for 1 year;

$\frac{24}{12} : \frac{11}{12} :: \$5460 : \$2502.50$, A's gain;
$\frac{24}{12} : \frac{13}{12} :: \$5460 : \$2957.50$, B's "

84. $2\frac{1}{2}$ B's + B's = $3\frac{1}{2}$ B's = capital 1st year;
$1\frac{1}{2}$ B's + B's = $2\frac{1}{2}$ B's = " 2d "

$3\frac{1}{2} : 2\frac{1}{2} :: \$1200$: A's gain 1st yr., = $\$857\frac{1}{7}$
$2\frac{1}{2} : 1\frac{1}{2} :: \$1200$: A's gain 2d yr., = \$720
A's whole gain = $\$1577\frac{1}{7}$

$\$2400 - \$1577\frac{1}{7} = \$822\frac{6}{7}$, B's whole gain.

Or, B's gain may be obtained by proportion.

85. \$5000 × 1 = \$5000
\$6000 × 1 = \$6000
\$7000 × 1 = \$7000
\$18000, A's capital for 1 year.

\$7000 × 1 = \$ 7000
\$5000 × 2 = 10000
\$17000, B's capital for 1 year.

\$18000 + \$17000 = \$35000, entire capital for 1 year.

\$35000 : \$18000 :: \$1750 : \$900, A's loss;
\$35000 : \$17000 :: \$1750 : \$850, B's "

Or, $\$1750 \times \frac{18}{35} = \$900$, A's loss;
$\$1750 \times \frac{17}{35} = \$850$, B's loss.

Page 245. (Continued.)

86. \$1.75 × 80 = \$140, amount he might have received;
\$140 — \$80 = \$60, amount lost;

\$ 60 ÷ \$2.50 = 24, days he was idle;
80 — 24 = 56, days he worked.

Had he worked every day he would have received \$1.75 × 80 = \$140, but he received \$60 (\$140 — \$80) less than this. Each day he was idle he lost his pay, \$1.75, and his board, \$.75, for that day, making a total loss of \$2.50 for each idle day. The \$60, which he lost owing to idleness, divided by \$2.50, the loss for each idle day, gives 24, the number of days he was idle; whence 80 — 24 = 56, number of days he worked.

87. $\frac{3}{10}$ of 1st × $\frac{3}{1}$ = $\frac{9}{10}$ of 1st = 2d carpet;
$\frac{3}{10}$ of 1st × $\frac{8}{3}$ = $\frac{24}{30}$, or $\frac{8}{10}$, of 1st = 3d carpet;

$\frac{10}{10} + \frac{9}{10} + \frac{8}{10} = \frac{27}{10}$; $\frac{27}{10}$ of 1st = 135 yards;

135 yards ÷ $\frac{27}{10}$ = 50 yards, 1st carpet;
50 yards × $\frac{9}{10}$ = 45 yards, 2d carpet;
50 yards × $\frac{8}{10}$ = 40 yards, 3d carpet.

EVOLUTION.

Page 254.

30. $\sqrt{\frac{289}{324}} = \frac{17}{18}$; $\sqrt{\frac{625}{2304}} = \frac{25}{48}$.

Since the square of a fraction is found by squaring each term, the square root of a fraction is found by extracting the square root of each term.

Page 254. (Continued.)

31. $272\frac{1}{4} = \frac{1089}{4}$; $\sqrt{\frac{1089}{4}} = \frac{33}{2} = 16\frac{1}{2}$.

Or, $272\frac{1}{4} = 272.25$; $\sqrt{272.25} = 16.5 = 16\frac{1}{2}$.

(2.) $1040\frac{1}{16} = \frac{16641}{16}$; $\sqrt{\frac{16641}{16}} = \frac{129}{4} = 32\frac{1}{4}$.

32. $\sqrt{2} = 1.414213562373095+$.

For the purpose of testing accuracy in the extraction of square root to a number of places, the square root of 2 to 15 decimal places is given.

Page 259.

18.

$$\begin{array}{rrl}
 & & 51\dot{5}84\dot{9}60\dot{8}(802 \\
8^3 = & & 512 \\
8^2 \times 3 = & 192) & \overline{\quad 38} \\
80^3 = & & \overline{512000} \\
80^2 \times 3 = & 19200) & \overline{\quad 38496} \\
802^3 = & & \overline{515849608}
\end{array}$$

20.

$$\begin{array}{rrl}
 & & 2\dot{7}05\dot{4}03\dot{6}00\dot{8}(3002 \\
3^3 = & & 27 \\
3^2 \times 3 = & 27) & \overline{\quad 00} \\
30^3 = & & \overline{27000} \\
30^2 \times 3 = & 2700) & \overline{\quad 540} \\
300^3 = & & \overline{27000000} \\
300^2 \times 3 = & 270000) & \overline{\quad 540360} \\
3002^3 = & & \overline{27054036008}
\end{array}$$

Page 260.

27. $\frac{216}{1728} = \frac{1}{8}$; $\sqrt[3]{\frac{1}{8}} = \frac{1}{2}$; (2.) $\sqrt[3]{\frac{1331}{42875}} = \frac{11}{35}$.

Page 260. (Continued.)

Since the cube of a fraction is found by cubing each term, the cube root of a fraction is found by extracting the cube root of each term.

28. $11\frac{25}{64} = \frac{729}{64}$; $\sqrt[3]{\frac{729}{64}} = \frac{9}{4} = 2\frac{1}{4}$.

Or, $11\frac{25}{64} = 11.390625$; $\sqrt[3]{11.390625} = 2.25 = 2\frac{1}{4}$.

(2) $37\frac{1}{27} = \frac{1000}{27}$; $\sqrt[3]{\frac{1000}{27}} = \frac{10}{3} = 3\frac{1}{3}$.

31. 2150.4 (number cubic inches in 1 bushel) $\times\ 100 =$ 215040, cubic inches;

$\sqrt[3]{215040} = 59.9 +$.

32. $128 \times 3 \times 6 \times 6 = 13824$, cubic feet;

$\sqrt[3]{13824} = 24$, feet high.

Page 265.

4. The length may be considered the base, and the width the perpendicular of a right-angled triangle of which the diagonal will be the hypotenuse.

$192^2 = 36864$; $144^2 = 20736$;
$36864 + 20736 = 57600$;
$\sqrt{57600} = 240$, diagonal.

6. $96^2 = 9216$; $72^2 = 5184$;
$9216 + 5184 = 14400$; $\sqrt{14400} = 120$.

8. $240^2 = 57600$; $208^2 = 43264$;
$57600 - 43264 = 14336$, feet;
$\sqrt{14336} = 119.73 +$.

9. $84 \times 63 = 5292$, square rods;
$\sqrt{5292} = 72.74 +$.

Page 265. (Continued.)

10. $125^2 = 15625$, square rods;
$15625 \div 150 = 104\frac{1}{6}$.

Page 266.

14. $3^2 : 24^2 :: 1 :$ Ans.; $1 \times 576 \div 9 = 64$, Ans.

Since the areas of circles are to each other as the squares of their diameters (Complete Arithmetic, Art. 421, Prin. 2), the circle whose diameter is 3 inches is to the circle whose diameter is 24 inches (2 feet) as 3^2 is to 24^2, or as 9 to 576, or (dividing both terms of the ratio by 9) as 1 to 64; that is, the first circle is $\frac{1}{64}$ the area of the second. Hence, it will take 64 of the first to equal in area the second.

15. $15^2 : 60^2 :: 1 :$ Ans.; $1 \times 3600 \div 225 = 16$, Ans.

Since the diameters are to each other as 15 inches to 60 inches, or as 1 to 4, the areas are as 1^2 to 4^2, or as 1 to 16; hence it will take 16 circles, 15 inches in diameter, to equal in area a circle 5 feet in diameter.

16. $80^2 \times .7854 = 5026.56$ sq. ft. $= 558.5+$ sq. yds.

The rope being 40 feet long, the diameter of the circular plat upon which the horse can graze will be 80 feet.

17. $218\frac{1}{6}$ sq. yds. $= 1963.5$ sq. ft.;

$1963.5 \div .7854 = 2500$, the square of the diameter; $\sqrt{2500} = 50$, the diameter; $50 \div 2 = 25$, the radius; hence 25 feet is the extreme distance from the stake that the horse can graze.

18. $3^2 = 9$; $9 \times .7854 = 7.0686$, sq. in. in circle;

Page 266. (Continued.)

$2.5 \times 2.5 = 6.25$, sq. ft. in square; 6.25 sq. ft. $= 900$ sq. in.; $900 \div 7.0686 = 127.32 +$, number of circles 3 inches in diameter, equal in area to a surface 2.5 ft. square.

Note.—The number of circles 3 inches in diameter which can be *inscribed* in a surface 2.5 feet square is 10×10, or 100.

Page 267.

23. $2^3 : 8^3 :: 1 :$ Ans.; $1 \times 512 \div 64$, Ans.

Since their diameters are as 2 to 8, or 1 to 4, their volumes are as 1^3 to 4^3, or as 1 to 64, and hence it will take 64 of the first to equal in volume the second.

24. The diameters being as 4 to 1, the volumes are as 4^3 to 1^3, or as 64 to 1. The earth is 64 times the moon in volume.

Note.—This answer is not absolutely correct, inasmuch as the diameters are not exactly as 4 to 1.

25. The diameters being as 85000 to 850000, or as 1 to 10, the volumes are as 1^3 to 10^3, or as 1 to 1000. The sun is 1000 times the planet Jupiter in volume.

26. $28274400 \div 3.1416 = 9000000$, square of diameter;
$\sqrt{9000000} = 3000$, diameter.

28. $904.7808 \div .5236 = 1728$; $\sqrt[3]{1728} = 12$.

29. $36.1 \times \frac{5}{2} = 90.25$, sq. in. in triangle;
$\sqrt{90.25} = 9.5$, side of equivalent square in inches

The area of a triangle equals one-half the product of the base by the altitude. See Complete Arith., Art. 161, 3.

Page 267. (Continued.)

31. 94.25 ft. $\div$ 3.1416 = 30.0006 ft., diameter;
$(30.0006)^2 \times .7854 = 706.88827$, area of section;
706.88827 sq. ft. × 100 = 70688.827, cu. ft.

32. $91400000 \times 2 = 182800000$, the diameter of the earth's orbit;

$182800000 \times 3.1416 = 574284480$, circumference of the earth's orbit;

$574284480 \div 8766$, the number of hours in $365\frac{1}{4}$ days, $= 65512.7+$, miles for the earth's mean hourly motion.

33. $35400000 \times 2 = 70800000$; $70800000 \times 3.1416 = 222425280$; $222425280 \div 2109.6 = 105434.8+$, miles.

34. $(2000)^2 \times 3.1416 =$ moon's surface;
$(8000)^2 \times 3.1416 =$ earth's surface;

$$\frac{(8000)^2 \times 3.1416}{(2000)^2 \times 3.1416} = 16.$$

Or, geometry shows that the surfaces of spheres are to each other as the squares of their diameters; the diameters of the earth and moon are as 8000 to 2000, or as 4 to 1, and the surfaces are as 4^2 to 1^2, or as 16 to 1; that is, the moon's surface is $\frac{1}{16}$ of the earth's surface.

GENERAL REVIEW.

Page 269.

17. If he sold $\frac{2}{3}$ of his sheep, he had $\frac{1}{3}$ left; and if he bought $\frac{3}{5}$ of $\frac{1}{3}$, or $\frac{1}{5}$, he then had $\frac{1}{3} + \frac{1}{5}$, or $\frac{8}{15}$, of the number he had at first; if $\frac{8}{15}$ of the number was 40, $\frac{1}{15}$ was 5 and $\frac{15}{15}$, or the whole number, was 75.

Page 269. (Continued.)

19. If $60 was $\frac{4}{5}$ of $\frac{3}{4}$, or $\frac{3}{5}$, of the cost; he lost $\frac{5}{5} - \frac{3}{5}$, or $\frac{2}{5}$, of the cost; if $60 is $\frac{3}{5}$ of the cost, $\frac{1}{5}$ is $20 and $\frac{2}{5}$, or the loss, is twice $20, or $40.

21. If B sold the horse for $\frac{1}{6}$ less than its cost, he received $\frac{5}{6}$ of its cost; if $80 is $\frac{5}{6}$ of its cost to B, $\frac{1}{6}$ is $16 and $\frac{6}{6}$, or the cost, is $96, which is $\frac{1}{5}$ more than it cost A; if $96 is $\frac{6}{5}$ of the cost to A, $\frac{1}{5}$ is $16 and $\frac{5}{5}$, or the cost, is $80: A paid $80 for the horse.

23. If $\frac{3}{4}$ of a bushel cost $\$\frac{3}{5}$, $\frac{1}{4}$ cost $\$\frac{1}{5}$ and $\frac{4}{4}$, or a bushel, cost $\$\frac{4}{5}$; at $\$\frac{4}{5}$ a bushel as much wheat can be bought for $\$\frac{5}{8}$ as $\$\frac{4}{5}$ is contained times in $\$\frac{5}{8}$, which is $\frac{25}{32}$: $\frac{25}{32}$ of a bushel can be bought for $\$\frac{5}{8}$.

Page 270.

32. If the loaf weighs 10 oz. when flour is $4 per barrel, it should weigh 4 times 10 oz., or 40 oz., when flour is $1 per barrel, and $\frac{1}{5}$ of 40 oz., or 8 oz., when flour is $5 per barrel.

35. If 3 men can mow 18 acres in 4 days, 1 man can mow 6 acres in 4 days, and $\frac{6}{4}$ of an acre, or $1\frac{1}{2}$ acres, in 1 day, and $4\frac{1}{2}$ acres in 3 days; if 1 man can mow $4\frac{1}{2}$ acres in 3 days, it will take as many men to mow 9 acres in 3 days as $4\frac{1}{2}$ acres are contained times in 9 acres, which is 2: it will take 2 men to mow 9 acres in 3 days.

39. Since the shorter part was $\frac{3}{5}$ of the longer, the length of both parts, or the pole, was equal to $\frac{5}{5}$ plus $\frac{3}{5}$, or $\frac{8}{5}$, of the longer part; if $\frac{8}{5}$ of the longer part was 120 feet, $\frac{1}{5}$ was 15 feet, and $\frac{5}{5}$, or the longer part, was 75 feet: the length of the shorter part was $\frac{3}{5}$ of 75 feet, which is 45 feet.

Page 270. (Continued.)

41. If B pays $\frac{1}{2}$ as much as A, and C $\frac{1}{2}$ as much as B, C pays $\frac{1}{4}$ as much as A, and the three together pay $\frac{4}{4}$ plus $\frac{2}{4}$ plus $\frac{1}{4}$, or $\frac{7}{4}$, as much as A; if $\frac{7}{4}$ of what A pays is $42, $\frac{1}{4}$ is $6, and $\frac{4}{4}$ is $24: A pays $24, B $\frac{1}{2}$ of $24, or $12, and C $\frac{1}{2}$ of $12, or $6.

43. If $\frac{3}{4}$ of B's money is $\frac{2}{3}$ of $81, which is $54, $\frac{1}{4}$ of B's money is $18, and $\frac{4}{4}$ is $72; if $\frac{3}{4}$ of A's money is $\frac{2}{3}$ of $72, which is $48, $\frac{1}{4}$ of A's money is $16, and $\frac{4}{4}$ is $64. Therefore A has $64 and B $72.

45. If A makes a shoe in $\frac{2}{3}$ of a day, he will make at the rate of $\frac{1}{2}$ a shoe in $\frac{1}{3}$ of a day, and $\frac{3}{2}$, or $1\frac{1}{2}$, shoes in a day. B makes at the rate of $\frac{1}{2}$ a shoe in $\frac{1}{5}$ of a day, and $\frac{5}{2}$, or $2\frac{1}{2}$, shoes in a day. Both will make $1\frac{1}{2} + 2\frac{1}{2}$, or 4, shoes in a day.

46. If A can mow an acre in $\frac{3}{4}$ of a day, in 1 day he can mow $\frac{4}{3}$ of an acre, and if B can mow an acre in $\frac{3}{5}$ of a day, in 1 day he can mow $\frac{5}{3}$ of an acre, and in 1 day A and B together can mow $\frac{4}{3}$ plus $\frac{5}{3}$, or $\frac{9}{3}$, of an acre, or 3 acres; if they can mow 3 acres in a day, it will take as long to mow 1 acre, as 3 acres are contained times in 1 acre, which is $\frac{1}{3}$; they can mow an acre in $\frac{1}{3}$ of a day.

Page 271.

48. Since both can build $\frac{1}{12}$ of it in a day, and A $\frac{1}{20}$ of it in a day, C can build $\frac{1}{12} - \frac{1}{20}$, or $\frac{1}{30}$, of it in a day; and it will take him 30 days to build it alone.

50. If A, B, and C can do a job in 20 days, they can do $\frac{1}{20}$ of it in 1 day; if A and B can do it in 40 days, they can do $\frac{1}{40}$ of it in 1 day; and if A and C can do it in 30 days,

Page 271. (Continued.)

they can do $\frac{1}{30}$ of it in 1 day; if A, B, and C can do $\frac{1}{20}$ of it in a day and A and B $\frac{1}{40}$ of it, C alone can do $\frac{1}{20}$ — $\frac{1}{40}$, or $\frac{1}{40}$, of it; if A, B, and C can do $\frac{1}{20}$ and A and C $\frac{1}{30}$, B alone can do $\frac{1}{20}$ — $\frac{1}{30}$, or $\frac{1}{60}$, of it; if A and B can do $\frac{1}{40}$ and B alone $\frac{1}{60}$, A can do $\frac{1}{40}$ — $\frac{1}{60}$, or $\frac{1}{120}$, of it; if A can do $\frac{1}{120}$ of the job in 1 day, B $\frac{1}{60}$ of it, and C $\frac{1}{40}$ of it, A can do the job in 120 days, B in 60 days, and C in 40 days.

53. To gain 33⅓%, or $\frac{1}{3}$, he must sell the cloth for $\frac{4}{3}$ of $90, which is $120; if he sells 40 yards for $120, he sells at $3 a yard.

56. If he gains 25%, or $\frac{1}{4}$, he sells the cloth for $\frac{5}{4}$ of the cost; if $2.50 is $\frac{5}{4}$, $\frac{1}{4}$ is $.50, and the cost is $2; to lose 20%, or $\frac{1}{5}$, he must sell at $\frac{4}{5}$ of $2, or $1.60 a yard.

57. If $12.50 is 33⅓%, or $\frac{1}{3}$, of what he earned, the amount earned was 3 times $12.50, or $37.50.

58. If he gained 50%, or $\frac{1}{2}$, he sold the watch for $\frac{3}{2}$ of its cost; if $90 was $\frac{3}{2}$, the cost was $60: if he had sold it for $75 he would have gained $75 — $60, or $15, which is $\frac{15}{60}$, or $\frac{1}{4}$, or 25%, of the cost.

59. If $\frac{2}{3}$ of selling price is $\frac{3}{4}$ of cost, $\frac{1}{3}$ of selling price is $\frac{1}{2}$ of $\frac{3}{4}$, or $\frac{3}{8}$, of cost, and $\frac{3}{3}$, or selling price, is $\frac{9}{8}$ of cost, which is 112½% of cost; hence the gain per cent is 12½.

60. If A lost 33⅓%, or $\frac{1}{3}$, of his capital, he then had left $\frac{2}{3}$; if $\frac{2}{3}$ of A's capital was $600, $\frac{1}{3}$ was $300 and his entire capital was $900: if B gained 50%, or $\frac{1}{2}$, of his capital, he then had $\frac{3}{2}$; if $\frac{3}{2}$ of B's capital was $600, $\frac{1}{2}$ was $200 and his entire capital was $400.

Page 271. (Continued.)

61. If the miller takes 10% as toll, I can bring away 100% less 10%, or 90%, or $\frac{9}{10}$, of the amount carried; if $\frac{9}{10}$ is 2 bushels, $\frac{1}{10}$ is $\frac{2}{9}$ of a bushel, and $\frac{10}{10}$, the amount carried, is $\frac{20}{9}$, or $2\frac{2}{9}$, bushels.

Page 272.

65. By the conditions, 5 times $\frac{7}{45}$ of B's income, or $\frac{7}{9}$ of it, is $50 more than $\frac{2}{3}$ of B's income, and hence $\frac{7}{9} - \frac{2}{3}$, or $\frac{1}{9}$, of B's income is $50, and the whole income is 9 times $50, or $450.

66. Since D travels 7 miles to gain 3 miles on C, he must travel as many times 7 miles to overtake C as 3 miles are contained times in 18 miles, which is 6; and 6 times 7 miles are 42 miles, the distance D travels to overtake C.

67. In 5 days he will travel 5 times 14 hours, or 70 hours; if he performs half the journey in 70 hours, to perform the other half will take as many days of 10 hours each as 10 is contained times in 70, which is 7: it will take 7 days.

68. $9\frac{4}{5}$ days of 8 hours each equal $9\frac{4}{5}$ times 8 hours, or $78\frac{2}{5}$ hours; working 6 hours a day, it will take as many days to do the same work as 6 hours is contained times in $78\frac{2}{5}$ hours, which is $13\frac{1}{15}$: it will take $13\frac{1}{15}$ days.

69. Since A is 20 years old and the sum of B's and C's ages is 4 times 20 years, or 80 years, the sum of A's, B's, and C's ages is 100 years; and since C's age is $\frac{1}{9}$ of A's and B's, $\frac{9}{9}$ plus $\frac{1}{9}$, or $\frac{10}{9}$, of A's and B's, is 100 years, and $\frac{1}{9}$ of A's and B's ages is 10 years and the sum of their ages is 90 years; B's age is 90 years minus 20 years, or 70 years, and C's is $\frac{1}{9}$ of 90 years, or 10 years.

Page 272. (Continued.)

70. Since the hound gains 2 rods in running 7 rods, he gains 1 rod in running $3\frac{1}{2}$ rods, and to gain 30 rods he must run 30 times $3\frac{1}{2}$ rods, or 105 rods.

71. Since 2 leaps of the hound equal 4 leaps of the hare, 1 leap of the hound equals 2 leaps of the hare, and hence the 50 leaps of the hare, the distance to be gained by the hound, are equal to 25 leaps of the hound, and the 4 leaps which the hare makes while the hound makes 3, are equal to 2 leaps of the hound; hence the hound makes 3 leaps to gain 1 on the hare, and, to catch the hare, he must make 3 times 25 leaps, or 75 leaps.

72. Since the steamer goes 9 miles down stream in 1 hour, it will be $\frac{1}{9}$ of an hour in going 1 mile down stream, and since it goes up stream at the rate of 5 miles an hour, it will be $\frac{1}{5}$ of an hour in going 1 mile up stream; and hence to go 1 mile down stream and back, it will take $\frac{1}{9}$ plus $\frac{1}{5}$, or $\frac{14}{45}$, of an hour, and so it can go as many miles down stream and back in 14 hours as $\frac{14}{45}$ of an hour is contained times in 14 hours, which is 45: it can go 45 miles down stream and return.

73. The steamer can go down the stream 1 mile and return in 5 minutes + 7 minutes, or 12 minutes, and in 1 hour, or 60 minutes, it can go and return as many miles as 12 minutes are contained times in 60 minutes, which is 5: it can go 5 miles and return.

Page 273.

74. Since the first pipe fills the cistern in 4 hours, it fills $\frac{1}{4}$ of it in 1 hour, and since the second pipe empties it in 6 hours, it empties $\frac{1}{6}$ of it in 1 hour, and when both pipes run, the quantity remaining in the cistern each hour is the

Page 273. (Continued.)

difference between the quantity running in, or $\frac{1}{4}$ of the capacity of the cistern, and the quantity running out, or $\frac{1}{6}$ of the capacity; the difference between $\frac{1}{4}$ and $\frac{1}{6}$ is $\frac{1}{12}$, and since $\frac{1}{12}$ of the cistern is filled each hour, it will take 12 hours to fill it.

75. At six o'clock the minute-hand is 30 spaces behind the hour-hand, and since the minute-hand gains on the hour-hand $\frac{11}{12}$ of a space each minute, it will take as many minutes for it to overtake the hour-hand as $\frac{11}{12}$ of a space is contained times in 30 spaces, which is $32\frac{8}{11}$; and hence the hands will be together at $32\frac{8}{11}$ minutes past six o'clock.

Or, since the minute-hand passes over $\frac{12}{11}$ of the distance gained on the hour-hand, it must pass over $\frac{12}{11}$ of 30 spaces, or $32\frac{8}{11}$ spaces; and the two hands will be together at $32\frac{8}{11}$ minutes past six o'clock.

78. $50 \times 4 = 200$, the remainder;
$200 + 64 = 264$, the product;
$264 \div 22 = 12$, the number.

89. $3 \div 8\frac{13}{16} = 3 \div \frac{141}{16} = \frac{48}{141} = \frac{16}{47}$.

90. $640 \times \frac{9}{16} = 360$; $360 \times \frac{2}{3} = 240$; $360 \times \frac{1}{3} = 120$;
$\$10 \times 240 = \$2400$; $\$12 \times 120 = \$1440$;
$\$2400 + \$1440 = \$3840$; $\$3840 - \$2880 = \$960$, gain.

91. $\frac{7}{8} \times \frac{3}{5} = \frac{21}{40}$; $\$16800 \div \frac{21}{40} = \$32000$, value of ship;
$\frac{7}{8} - \frac{21}{40} = \frac{7}{20}$, part left.

Page 274.

103. $29 \times 24 \times 60 = 41760$, minutes.

Since 1880 is divisible by 4, it is a leap-year, and February has 29 days.

108. $18\frac{1}{2} \times 15 \div 9 = 30\frac{5}{6}$, sq. yds. in floor;
$30\frac{5}{6} \div \frac{3}{4} = 41\frac{1}{9}$, yards of carpeting.

A yard of carpeting $\frac{3}{4}$ of a yard wide will cover $\frac{3}{4}$ of a square yard of surface.

Page 275.

126. $100\% - 8\% = 92\%$; $92\% \times \frac{3}{4} = 69\%$, mustered;
$621 \div .69 = 900$, number of men in regiment at first.

129. $\$24 \div .75 = \$32$, cost;
$\$34 - \$32 = \$2$, amount gained;
$\$2 \div \$32 = .06\frac{1}{4} = 6\frac{1}{4}\%$.

Page 276.

130. $100\% + 20\% = 120\%$; $120\% \times .20 = 24\%$;
$120\% - 24\% = 96\%$; $100\% - 96\% = 4\%$, loss.

The first lot of goods was sold for 120% of cost, and the second lot was sold at 20% loss, which was 24% of the cost of the first lot of goods, and hence the second lot of goods was sold for 96% of the cost of the first lot, giving a loss of 4% on the whole transaction.

131. $\$240 \div 1.20 = \$200$, cost of first;
$\$240 \div .80 = \$300$, cost of second;
$\$200 + \$300 = \$500$, cost of both;
$\$500 - \$480 = \$20$, loss.

Page 276. (Continued.)

132. $72 × 1.25 = $90, selling price;
$90 ÷ .90 = $100, price asked.

133. 130% × .10 = 13%; 30% — 13% = 17%, gain;
$5800 × .17 = $986, amount gained.

It is to be noticed that the per cent of gain is obtained without regard to the value of the goods involved.

134. $4 × 1.20 = $4.80, selling price;
$4.80 ÷ .80 = $6.00, price asked.

To gain 20%, I must sell the cloth for 120% of $4, which is $4.80, and since the selling price is to be 20% less than the price asked, $4.80 is 80% of the price asked.

135. 120% — 75% = 45%; 45% ÷ 75% = .60 = 60%.

He bought at 75% and sold at 120% of the par value, gaining 120% — 75%, or 45% of the par value, which is $\frac{45}{75}$, or 60%, of his investment.

136. 100% — $33\frac{1}{3}$% = $66\frac{2}{3}$%; $66\frac{2}{3}$% × 1.50 = 100%.

He lost $33\frac{1}{3}$% of the apples, and sold $66\frac{2}{3}$% at 150% of the cost, which was 100% of the cost of the whole lot: hence he neither gained nor lost.

137. $\$.04\frac{1}{2}$ × 100 × 63 = $283.50, cost;
$283.50 × $\frac{2}{3}$ = $189.00, cost of $\frac{2}{3}$;
$283.50 × $\frac{1}{2}$ = $141.75, cost of $\frac{1}{2}$;
$189.00 — $141.75 = $47.25, loss on $\frac{2}{3}$ sold;
$47.25 ÷ $189.00 = .25 = 25%.

Or, $\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$, part of cost lost;
$\frac{1}{6} \div \frac{2}{3} = \frac{1}{4} = .25 = 25\%$.

It is to be noted that here, as in all other cases, the *base* is the cost of the goods sold.

Page 276. (Continued.)

138. $128.25 × 1.04 = $133.38, amount of $128.25 for 6 months, at 8%;
$133.38 × 1.06 = $141.3828, selling price.

139. $800 × 1.03 = $824, amt. of $800 for 4 mo., at 9%;
$959.10 — $824 = $135.10, net gain;
$135.10 ÷ $800 = $.16\frac{71}{80}$ = $16\frac{71}{80}$%.

140. $3240 × $\frac{2}{3}$ = $2160; $2160 × $.00\frac{3}{4}$ = $16.20.

141. $19.20 ÷ .015 = $1280, amount insured;
$1280 ÷ $\frac{5}{8}$ = $2048, value of house.

Page 277.

142. $25000 ÷ 1.025 = $24390.2439+, proceeds to be invested in cotton;
$24390.2439 ÷ $.05 = 487804.87, pounds.

143. $502.50 ÷ 1.005 = $500, the amount invested;
$500 ÷ $1.25 = 400, number of yards bought.

147. $800 × .15 ÷ 6 = $20, interest at 1% for 2 yr. 6 mo.;
$90 ÷ $20 = $4\frac{1}{2}$, rate.

148. $337.40 — $311.50 = $25.90, interest;
$311.50 × .08 ÷ 6 = $\$4.15\frac{1}{3}$, int. at 1% for 1 yr. 4 mo.;
$ 25.90 ÷ $\$4.15\frac{1}{3}$ = $6\frac{21}{89}$, rate.

149. $540 × $.11\frac{2}{3}$ = $63; $63 ÷ .135 = $\$466\frac{2}{3}$.

Page 277. (Continued.)

150. $\$750 \div \frac{5}{4} = \$600$, principal;
$\$750 - \$600 = \$150$, interest for 3 years;
$\$150 \div 3 = \$50$, interest for 1 year;
$\$50 \div \$600 = .08\frac{1}{3} = 8\frac{1}{3}\%$.

151. 1865 11 4 $500.
1864 10 8 $.064\frac{1}{3}$
1 yr. 0 mo. 26 da. $.064\frac{1}{3}$. 2)$\$32.166\frac{2}{3}$
$30. $16.083\frac{1}{3}$
$48.25, 1st int.
1866 1 30 $500
1865 11 4 $.014\frac{1}{3}$
2 mo. 26 da. $.014\frac{1}{3}$. 2)$\$7.166\frac{2}{3}$
$250. $3.583\frac{1}{3}$
$10.75, 2d int.
1866 7 1 48.25, 1st int.
1866 1 30 $500.
5 mo. 1 da. $.025\frac{1}{6}$. $559. [payt.
280 1st and 2d
$279.
$.025\frac{1}{6}$
2)$7.0215
3.5107
$10.5322, 3d int.
279.
$289.532, amt. due.

152. $\$1.00 + \$.20 = \$1.20$, amt. of $1 for 3 yr. 4 mo.;
$\$1320 \div 1.20 = \$1100$, present worth;
$\$1320 - \$1100 = \$220$, discount.

Page 277. (Continued.)

155. $\$359.50 \div 1.03 = \$349.029+$, present worth;
$\$359.50 - \$349.029 = \$10.471$, discount;
$\$359.50 \times .03 = \$10.785$, bank discount;
$\$10.785 - \$10.471 = \$.314$, difference.

156. $\$1.00 - .034\frac{1}{6} = \$.965\frac{5}{6}$, proceeds of \$1, discounted for 4 months 3 days, at 10%; $\$300 \div .965\frac{5}{6} = \$310.61+$.

Page 278.

159.
$$\begin{array}{rrl} & \$\ 300 \times 5 = & \$1500 \\ & \$\ 700 \times 3 = & \$2100 \\ & \underline{\$\ 200} \times 8 = & \underline{\$1600} \\ 2) & \$1200 & \$5200 \\ & \underline{\$\ 600} \times 2 = & \underline{\$1200} \\ & \$\ 600 &)\$4000(6\frac{2}{3} \end{array}$$

The equated time for the payment of the other half is $6\frac{2}{3}$ months.

Or, $\$5200 \div \$1200 = 4\frac{1}{3}$, the equated time for the payment of the whole; $4\frac{1}{3}$ months $-$ 2 months $= 2\frac{1}{3}$ months; since one-half is paid $2\frac{1}{3}$ months before maturity, the other half may be kept $2\frac{1}{3}$ months after maturity, or $4\frac{1}{3}$ months $+$ $2\frac{1}{3}$ months, or $6\frac{2}{3}$ months.

160. 50 bu. $\times$ 1.20 = 60 bu.;
$\$150 \div 2 = \$75$, A's share;
$\$150 \div 2 = \$75$, B's share.

Since a bushel of A's wheat is worth 20% more than a bushel of B's, A's 50 bushels equal in value B's 60 bushels, and hence each should receive $\frac{1}{2}$ of the \$150, or \$75.

Page 278. (Continued.)

161. $5\frac{1}{2} + 8 = 13\frac{1}{2}$, number of cords;
$\$27 \div 13\frac{1}{2} = \$2$, cost per cord;
$\$2 \times 5\frac{1}{2} = \$11$, amount to be paid by first;
$\$2 \times 8 = \$16$, amount to be paid by second.

162. 84 cords $\div 12 \div 7 \div 8 = \frac{1}{8}$ cord, what 1 man can cut in 1 hour;
$\frac{1}{8}$ cord $\times 10 \times 5 = 6\frac{1}{4}$ cords, what 1 man can cut;
150 cords $\div 6\frac{1}{4}$ cords $= 24$, number of men.

Or, $\left.\begin{array}{rcr} 84 & : & 150 \\ 10 & : & 12 \\ 5 & : & 7 \end{array}\right\} :: 8 \text{ men} : \text{Ans.} = 24 \text{ men.}$

165. $\$53\frac{2}{3} \div 7\frac{2}{3} \div 4 = \$1\frac{3}{4}$, amt. earned by 1 man in 1 day;
$\$1\frac{3}{4} \times 7 \times \frac{4}{49} = \$1.$

Or, $\left.\begin{array}{rcr} 4 & : & 7 \\ 7\frac{2}{3} & : & \frac{4}{49} \end{array}\right\} :: \$53\frac{2}{3} : \text{Ans.} = \$1.$

166. $\$750 - \$600 = \$150$, A's gain;
$\$600 \div \$150 = 4$,
$\$1200 \times 4 = \$4800$, B's stock.

Since B's gain ($600) was 4 times A's gain ($150), B's stock was 4 times A's stock, or 4 times $1200, which is $4800.

167. $\$2000 \times 12 = \$24000$, A's stock for 1 month;
$800 \times 12 = 9600$, B's " " " "

$$6\overline{)\$14400}, \text{ excess of A's stock for 1 month;}$$
$$\$2400$$

Since A's stock for 1 month is $14400 more than B's, B must put in for 6 months $\frac{1}{6}$ of $14400, or $2400.

Page 278. (Continued.)

$$\begin{array}{ll} \textbf{168.}\ \$200 \times 5 = & \$1000 \\ \quad\ \ \underline{\$300} \times 2 = & \underline{\$\ 600} \\ \quad\ \ \$500 & \$1600 \end{array}$$

$\$1389 - \$\ 500 = \$889$, whole gain;

$\$1000 \div \$1600 = \frac{5}{8}$, A's part of gain;
$\$\ 600 \div \$1600 = \frac{3}{8}$, B's part of gain;

$\$\ 889 \times \frac{5}{8} = \$555\frac{5}{8}$, A's gain;
$\$\ 889 \times \frac{3}{8} = \$333\frac{3}{8}$, B's gain;

$\$555\frac{5}{8} + \$\ 200 = \$755\frac{5}{8}$, A's share;
$\$333\frac{3}{8} + \$\ 300 = \$633\frac{3}{8}$, B's share.

171. $\sqrt{50^2 - 30^2} = \sqrt{1600} = 40$, feet.

The ladder is the *hypotenuse* of a right-angled triangle, the height of the window being the *altitude*, and the distance of the foot of the ladder from the base of the building, the *base*.

Page 279.

172. $\sqrt{52^2 + 39^2} = \sqrt{4225} = 65$, miles.

173. $\sqrt{40^2 + 30^2} = \sqrt{2500} = 50$, feet.

174. 10 acres $= 160$ sq. rds. $\times 10 = 1600$ sq. rds.;
$\sqrt{1600} = 40$, rods in each side of square;
$40 \times 4 = 160$, rods in perimeter of square.

176. $20^2 + 16^2 = 656$; $\sqrt{656} = 25.61+$, feet, distance from one corner of the floor to the opposite corner;

$(25.61+)^2 + 12^2 = 800$; $\sqrt{800} = 28.28+$, feet, distance from a lower corner to the opposite upper corner.

Page 279. (Continued.)

NOTE.—The student will perhaps understand the above process more clearly by supposing a lower corner to be called A, its opposite lower corner B, its opposite upper corner C. Then the line AB is 25.61+ feet, obtained from the right-triangle whose sides are 20 and 16, and AC is 28.28+ feet, obtained from the right-triangle whose sides are 25.61+ and 12. $(25.61+)^2 = (\sqrt{656})^2 = 656$.

178. $6 \times 4 = 24$, sq. ft.; $24 \times 2 = 48$, sq. ft. in top and bottom;
$6 \times 1\frac{1}{2} = 9$, sq. ft.; $9 \times 2 = 18$, sq. ft. in sides;
$4 \times 1\frac{1}{2} = 6$, sq. ft.; $6 \times 2 = 12$, sq. ft. in ends;
$48 + 18 + 12 = 78$, sq. ft. in whole surface.

Or, $6 \times 4 + 6 \times 1\frac{1}{2} + 4 \times 1\frac{1}{2} = 39$; $39 \times 2 = 78$.

Or, $6 \times 4 \times 2 + 6 \times 1\frac{1}{2} \times 2 + 4 \times 1\frac{1}{2} \times 2 = 78$.

179. $40 \times 3.1416 = 125.664$, rods in circumference;
$125.664 \times \frac{40}{4} = 1256.64$, square rods in field;
$1256.64 \div 160 = 7.854$, acres.

180. $470.875 \div .7854 = 599.52+$, the square of the diameter;
$\sqrt{599.53+} = 24.48+$.

181. $2^3 : 8^3 :: 1 :$ Ans. $= 64$.

Or, since the two diameters are to each other as 1 to 4, the larger ball weighs 64 times as much as the smaller.

182. $8 \times 4 \times 2 = 64$, cu. ft. in block of wood;
$(\frac{1}{3})^3 = \frac{1}{27}$, cu. ft. in small block;
$64 \div \frac{1}{27} = 1728$.

Page 279. (Continued.)

183. $8 \times 5 \times 4 \times 1728 = 276480$, cu. in.;
$276480 \div 2150.4$ (cu. in. in bushel) $= 128.57+$, bu.

184. $7 \times 6 \times 11 \times 1728 = 798336$, cu. in.;
$798336 \div 231$ (cu. inches in gal.) $= 3456$, gal.

185. $6 \times 4^2 \times .7854 \times 1728 = 130288.4352$, cu. in.;
$130288.4352 \div 231 = 564.019+$, gal.

186. B's share $+ \$95 =$ C's share;
B's share $+ \$95 + \$120 =$ A's share; hence,
B's $+$ B's $+ \$95 +$ B's $+ \$95 + \$120 = \$1000$;
3 times B's $+ \$310 = \$1000$;
3 times B's $= \$1000 - \$310 = \$690$;

B's share $= \$230$;
C's " $= \$230 + \$95 = \$325$;
A's " $= \$325 + \$120 = \$445$.

187. $2 \div 3 = \frac{2}{3}$, part of acre A can mow in 1 day;
$5 \div 6 = \frac{5}{6}$, " " " B " " " " "
$\frac{2}{3} + \frac{5}{6} = \frac{3}{2}$, " " " both " " " " "
$9 \div \frac{3}{2} = 6$, days required to mow 9 acres.

Page 280.

188. $\$726 \div 1.10 = \$660$; $\$660 \div 1.10 = \$600$;
$\$600 \div 1.10 = \$545.454+$, cost to A.

Or, let $100\% =$ A's buying price;
then, $100\% \times 1.10 = 110\% =$ A's selling price;
$110\% \times 1.10 = 121\% =$ B's selling price;
$121\% \times 1.10 = 133.1\% =$ C's selling price;
$\$726 \div 1.331 = \$545.454+$.

Page 280. (Continued.)

189. 2 ft. 8 in. $= 32$ in.; 32 in. $\times 5 = 160$ in.;
1 ft. 10 in. $= 22$ in.; 22 in. $\times 8 = 176$ in.;
$176 \div 160 = \frac{176}{160} = \frac{11}{10}$; $3\frac{3}{4}$ miles $= \frac{15}{4}$ miles;
$\frac{15}{4}$ miles $\times \frac{11}{10} = \frac{33}{8}$ miles $= 4\frac{1}{8}$ miles.

Since the boy walks 176 inches while the man walks 160 inches, the distance walked by the boy is $\frac{176}{160}$, or $\frac{11}{10}$, of the distance walked by the man.

190. $\frac{14}{14} - \frac{5}{14} = \frac{9}{14}$; $\frac{9}{14} \times \frac{1}{3} = \frac{3}{14}$, youngest son's share;
$\frac{5}{14} - \frac{3}{14} = \frac{2}{14} = \frac{1}{7}$, excess of share of eldest over share of youngest;
$\$1300 \times 7 = \$9100$, value of whole estate;
$\$9100 \times \frac{5}{14} = \$3250$, share of eldest;
$\$9100 \times \frac{6}{14} = \$3900$, share of second;
$\$9100 \times \frac{3}{14} = \$1950$, share of youngest.

191. $\$3.13 \div 6 = \$.52\frac{1}{6}$, cost of bushel of oats;
$\$.52\frac{1}{6} \times \frac{14}{5} =$ cost of bushel of rye;
$\$.52\frac{1}{6} \times \frac{14}{5} \times \frac{10}{7} = \$.52\frac{1}{6} \times 4 = \$2.08\frac{2}{3}$, cost of 1 bushel of wheat;
$\$50 \div \$2.08\frac{2}{3} = 23\frac{301}{313}$, number of bushels of wheat.

Or, $\$3.13 \div 6 =$ cost of 1 bu. of oats;
$\$3.13 \div 6 \times 14 \div 5 =$ cost of 1 bu. of rye;
$\$3.13 \div 6 \times 14 \div 5 \times 10 \div 7 = \$2.08\frac{2}{3}$, cost of 1 bu. of wheat.
$\$50 \div \$2.08\frac{2}{3} = 23\frac{301}{313}$, number of bushels of wheat.

192. Number of women $+ 4 =$ number of men;
Twice number of women $+ 4 =$ number of men and women;
Twice number of women $+ 4 + 10 =$ number of children;
Four times number of women $+ 4 + 4 + 10 =$ number of men, women, and children;

Page 280. (Continued.)

Hence, four times number of women + 18 = 90;
Four times number of women = 90 − 18 = 72;
Number of women = 18;
Number of men = 18 + 4 = 22;
Number of children = 18 + 22 + 10 = 50.

193. $\frac{4}{4} + \frac{3}{4} = \frac{7}{4}$; $\frac{7}{4} \times \frac{1}{2} = \frac{7}{8}$; $\frac{7}{4} + \frac{7}{8} = \frac{21}{8}$;
$6.30 ÷ $\frac{21}{8}$ = $240, share of first;
$240 × $\frac{3}{4}$ = $180, share of second;
($240 + $180) × $\frac{1}{2}$ = $210, share of third.

Since the second has $\frac{3}{4}$ as much as the first, the first and second together have $\frac{7}{4}$ as much as the first; the third has $\frac{1}{2}$ of this, or $\frac{7}{8}$ as much as the first, and all three have $\frac{7}{4} + \frac{7}{8}$, or $\frac{21}{8}$, as much as the first.

194. $\frac{1}{12}$ of work is what A and B can do in 1 day;
$\frac{1}{9}$ " " " " B and C " " " " "
$\frac{1}{6}$ " " " " A and C " " " " "

$\frac{1}{12} + \frac{1}{9} + \frac{1}{6} = \frac{13}{36}$, twice what all can do in 1 day;
$\frac{13}{36} \div 2 = \frac{13}{72}$, what all can do in 1 day;
$\frac{13}{72} - \frac{1}{12} = \frac{7}{72}$, what C can do in 1 day;
$1 \div \frac{7}{72} = 10\frac{2}{7}$, number of days in which C can do the work;
$\frac{13}{72} - \frac{1}{9} = \frac{5}{72}$, what A can do in 1 day;
$1 \div \frac{5}{72} = 14\frac{2}{5}$, number of days in which A can do the work;
$\frac{13}{72} - \frac{1}{6} = \frac{1}{72}$, what B can do in 1 day;
$1 \div \frac{1}{72} = 72$, number of days in which B can do the work.

195. $\frac{10}{10} - \frac{9}{10} = \frac{1}{10}$, what A does in $\frac{1}{2}$ day;
$\frac{1}{10} \div \frac{1}{2} = \frac{1}{5}$, " " can do " 1 "

Page 280. (Continued.)

$1 \div \frac{1}{5} = 5$, no. of days in which A can do the work;
$\frac{1}{5} \times 2 = \frac{2}{5}$, what A did in 2 days;
$\frac{9}{10} - \frac{2}{5} = \frac{1}{2}$, " B " " " ".
$\frac{1}{2} \div 2 = \frac{1}{4}$, " " " " 1 day;
$1 \div \frac{1}{4} = 4$, no. of days in which B can do the work.

196. 150% ÷ .60 = 250%; 250% of D's = $2\frac{1}{2}$ times D's = C's original capital.

Or, since 60% of C's original capital equaled 150% of D's, 1% of C's equaled $2\frac{1}{2}$% of D's, and the whole of C's equaled 250%, or $2\frac{1}{2}$ times D's.

197. $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$; $\frac{1}{6} \div \frac{1}{4} = \frac{2}{3}$; $\frac{3}{3} + \frac{2}{3} = \frac{5}{3}$;
$100 \div \frac{5}{3} = 60$, no. miles walked 1st day;
$60 \times \frac{2}{3} = 40$, " " " 2d "

Since $\frac{1}{4}$ of the second day's distance added to $\frac{1}{3}$ of the first gives $\frac{1}{2}$ of the first, $\frac{1}{4}$ of the second equals $\frac{1}{2} - \frac{1}{3}$, or $\frac{1}{6}$, of the first, and $\frac{4}{4}$ of the second equals $\frac{4}{6}$, or $\frac{2}{3}$, of the first, and hence $\frac{3}{3} + \frac{2}{3}$, or $\frac{5}{3}$, of the first equals 100 miles.

198. 15 ft. ÷ 2 = $7\frac{1}{2}$ ft., Ans.

Or, 2 : 1 :: 15 ft. : $7\frac{1}{2}$ ft., Ans.

It is a well known principle of mechanics, that when a body is supported at two points, the weight sustained at the two points respectively is *inversely* as their distance from the center of gravity, which, in the stick of timber, corresponds with its center, or is 15 feet from each end. The man at the end of the stick of timber is 15 feet from its center, and in order that the two men at the handspike may carry two-thirds of the stick, or twice as much as the

Page 280. (Continued.)

man at the end, the handspike must be placed one-half as far from the center of the stick, or one-half of 15 feet, which is $7\frac{1}{2}$ feet from the center, or $7\frac{1}{2}$ feet from the end.

Or, calling the weight supported at the end of the stick *one*, the weight supported at the handspike will be *two*, and we have the proportion: 2 : 1 :: 15 ft. : Ans.

This solution may be preferred:

According to a well known principle of physics, the product of one force by the distance of its point of application from the weight must equal the product of the other force by the distance of its point of application from the weight. The weight in this case must be considered as concentrated at the middle of the stick, its center of gravity. Calling the force exerted at the end of the stick *one*, the force at the handspike will be *two*, and we have the force *one* multiplied by 15 equal to the force *two* multiplied by its distance from the middle of the stick, which is 15 feet divided by 2, or $7\frac{1}{2}$ feet. Hence the handspike must be placed $7\frac{1}{2}$ feet from the end opposite the one man.

Page 281.

199. 4 : 8 :: 40 ft. : height of 1st tree = 80 ft.;
4 : 5 :: 40 ft. : height of 2d tree = 50 ft.;
80 ft. — 50 ft. = 30 ft., difference in the heights of the two trees.

$\sqrt{40^2 + 30^2} = \sqrt{2500} = 50$, number of feet between the tops.

A line drawn horizontally from the top of the second tree to the first tree will reach it 30 feet below its top, and a line joining the tops of the trees will be the hypotenuse of a right-triangle of which 40 feet, the width of the stream, is the *base*, and 30 feet, the *perpendicular*.

Page 281. (Continued.)

200. $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$, part flowing in in 1 hour;
$\frac{1}{5}+\frac{1}{6}+\frac{1}{7\frac{1}{2}}=\frac{1}{2}$, part flowing out in 1 hour;
$\frac{5}{6}-\frac{1}{2}=\frac{1}{3}$, part remaining in each hour;
$1 \div \frac{1}{3}=3$, no. of hours in which it will be filled.

A quantity of liquid equal to $\frac{5}{6}$ the capacity of the cistern flows in each hour, while a quantity equal to $\frac{1}{2}$ of its capacity flows out each hour; and hence the quantity in the cistern increases by their difference, $\frac{1}{3}$ the capacity of the cistern, each hour, which would fill the cistern in 3 hours.

APPENDIX.

Page 299.

3. $1+(4\times 11)=45$, first term.

Page 300.

6. $(41-21)\div 5=4$, com. difference;
25, 29, 33, 37, terms required.

7. $(177-12)\div 5=33$; $33+1=34$.

8. $(20+152)\div 2\times 45=3870$.

9. (6) $(41-21)\div 5=4$, com. difference;
$21-(4\times 3)=9$, first term;
$(9+41)\div 2\times 9=225$.

(7) $(177-12)\div 5=33$; $33+1=34$, no. of terms;
$(12+177)\div 2\times 34=3213$.

Page 300. (Continued.)

10. $(1 + 12) \div 2 \times 12 = 78$; $78 \times 2 = 156$.

The hammer of a clock makes 78 strokes in every 12 hours, and twice 78 strokes in 24 hours.

11. 2 cts. + 3 cts. × 49 = 149 cts.;
(2 cts. + 149 cts) ÷ 2 × 50 = 3775 cts = \$37.75.

Page 302.

2. $5 \times 3^6 = 3645$.

3. $1220 \times (\frac{1}{3})^5 = 5\frac{5}{243}$.

4. $64 \div 2^9 = \frac{64}{512} = \frac{1}{8}$.

5. (3) $(1220 \times 3 - 5\frac{5}{243}) \div 2 = 1827\frac{119}{243}$.

The descending series in the 3d problem equals an ascending series with $5\frac{5}{243}$ the first term, 1220 the last term, and 3 the common ratio.

6. $1215 \div 5 = 243$; $\sqrt[5]{243} = 3$.

7. $(2430 \times 3 - 10) \div 2 = 3640$.

8. $\$\frac{1}{2} \times 2^9 = \$\frac{1}{2} \times 512 = \$256$;
$(\$256 \times 2 - \$.50) \div 1 = \$511.50$.

9. $\$.01 \times 5^{14} = \$61035156.25$;
$(\$61035156.25 \times 5 - \$.01) \div 4 = \$76293945.31\frac{1}{4}$.

Page 305.

4.

$$150 \left|\begin{array}{l} 125 \\ 140 \\ 160 \\ 175 \end{array}\right. \quad \left.\begin{array}{r} 25 \\ 10 \\ 10 \\ 25 \end{array}\right\} \div 5 = \left\{\begin{array}{l} 5 \\ 2 \\ 2 \\ 5 \end{array}\right.$$

Page 305. (Continued.)

Or, since the gain on a pound of the first tea (25 cents) equals the loss on a pound of the fourth (25 cents), and the gain on a pound of the second tea (10 cents) equals the loss on a pound of the third (10 cents), the teas may be mixed in equal quantities, as one pound of each.

6.

$$65\left|\begin{array}{l}75\\60\\50\\40\end{array}\right.\qquad\left.\begin{array}{r}5+15+25=45\\ \text{- - - - - - -}\ 10\\ \text{- - - - - - -}\ 10\\ \text{- - - - - - -}\ 10\end{array}\right\}\div 5=$$

$$\left\{\begin{array}{r|l}9 & 9\text{ bu.}\times 30=270\text{ bu.}\\ 2 & 2\ \text{``}\ \times 30=\ \ 60\ \text{``}\\ 2 & 2\ \text{``}\ \times 30=\ \ 60\ \text{``}\\ 2 & 2\ \text{``}\ \times 30=\ \ 60\ \text{``}\end{array}\right.$$

$$60\text{ bu.}\div 2\text{ bu.}=30.$$

8.

$$18\left|\begin{array}{l}20\\17\\15\end{array}\right.\qquad\left.\begin{array}{r}1+3=4\\ \text{- - - -}\ 2\\ \text{- - - -}\ 2\end{array}\right\}\div 2=$$

$$\left\{\begin{array}{r|l}2 & 2\text{ lb.}\times 50=100\text{ lb.}\\ 1 & 1\ \text{``}\ \times 50=\ \ 50\ \text{``}\\ 1 & 1\ \text{``}\ \times 50=\ \ 50\ \text{``}\end{array}\right.$$

$$4;\quad 200\text{ lb.}\div 4\text{ lb.}=50.$$

Or, since the proportionals are 2, 1, 1, and their sum is 4, the 200 pounds may be divided into 4 parts, of 50 pounds each, 2 parts, or 100 pounds, being sugar at 20 cents, 1 part, or 50 pounds, sugar, at 17 cents, and 1 part, or 50 pounds, sugar, at 15 cents.

Page 305. (Continued.)

9. $$50\left|\begin{array}{l}60\,\urcorner\\0\,\lrcorner\end{array}\right.\quad\left.\begin{array}{r}50\\10\end{array}\right\}\div10=\left\{\begin{array}{r}5\\1\end{array}\right.\left|\begin{array}{l}5\times15=75,\text{ gal. vinegar.}\\1\times15=15,\text{ gal. water.}\end{array}\right.$$

Or, 50 cts. × 90 = 4500 cts., value of mixture;
4500 cts. ÷ 60 cts. = 75, gal. of vinegar;
90 — 75 = 15, gal. of water.

Page 308.

1. $1\times2\times3\times4\times5\times6=720$.

2. $1\times2\times3\times4\times5\times6\times7\times8\times9\times10\times11=39916800$.

3. $1\times2\times3\times4\times5\times6\times7\times8\times9=362880$.

4. $1\times2\times3\times4\times5\times6\times7\times8=40320$.

FOREIGN EXCHANGE.

Page 312.

2. £219 10s. 6d. = £219.525;
$\$4.91\frac{1}{9}\times219.525=\$1078.111$, cost in gold.

Page 313.

4. £300 8s. = £300.4;
$\$4.88\times300.4=\$1465.952$, cost in gold;
$\$1465.952\times1.14\frac{1}{4}=\$1674.85$, cost in currency.

Page 313. (Continued.)

5. $4.86 × 250 = $1215, gold cost in New York;
$1215 × 1.105 = $1342.575, currency cost in New York;
$1342.575 × 1.00⅛ = $1344.253, cost in Cincinnati.

Or, $4.86 × 1.00⅛ = $4.8660¾, cost of £1 in Cincinnati;
$4.8660¾ × 250 = $1216.51875, cost in gold;
$1216.51875 × 1.105 = $1344.253, cost in currency.

6. $4.855 × 1000 = $4855, cost in gold;
$4855 × 1.09¼ = $5304.087, cost in currency.

8. $2195.475 ÷ 1.00¼ = $2190, cost at quoted price;
$2190 ÷ $4.86⅔ = 450; £1 × 450 = £450.

Or, $4.86⅔ × 1.00¼ = $4.8788⅓, cost of £1;
$2195.475 ÷ $4.8788⅓ = 450; £1 × 450 = £450.

9. $4.89¼ × 1.10 = $5.38175, cost of £1 in currency;
$1291.62 ÷ 5.38175 = 240; £1 × 240 = £240.

Page 314.

2. 6475 fr. ÷ 5.18 fr. = 1250; $1 × 1250 = $1250.

3. 5330 fr. ÷ 5.125 fr. = 1040;
$1040 × 1.08⅜ = $1127.10.

Page 315.

5. $2584 ÷ 1.06¼ = $2432, value in gold;
5.16⅔ fr. × 2432 = 12565⅓ fr.

6. 6418¾ ÷ 5.135 = 1250; $1 × 1250 = $1250.

7. $741 ÷ 1.14 = $650, value in gold;
5.15 fr. × 650 = 3347½ fr.

Page 316.

2. $\$.96\frac{1}{2} \times 1680 \div 4 = \$405.30$, cost in gold;
$\$405.30 \times 1.13\frac{1}{2} = \$460.015$, cost in currency.

3. $\$.96\frac{1}{8} \times 800 = \$769$, value in gold;
$\$769 \times 1.10\frac{1}{2} = \$849.745$, value in currency;
$\$849.745 \times 1.00\frac{1}{4} = \$851.869$, cost.

5. $\$360.45 \div 1.11\frac{1}{4} = \$324$, in gold;
$\$324 \div \$.94\frac{1}{2} \times 4 = 1371\frac{3}{7}$, reichsmarks.

6. $\$.95\frac{3}{4} \times 240 = \$229.80$, cost in gold;
$\$229.80 \times 1.09\frac{1}{4} = \$251.056$, cost in currency.

7. $\$565.785 \div 1.10 = \$514.35$, gold;
$\$514.35 \div \$.95\frac{1}{4} \times 4 = 2160$, reichsmarks.

8. $\$238.6752 \div 1.00\frac{1}{4} = \$238.08$;
$\$238.08 \div 1.03\frac{1}{3} = \$230.40$, gold;
$\$230.40 \div \$.24 = 960$, reichsmarks.